

5.1 ~~5.1~~

Polynomial Long Division Notes

Key

#1

Warm-up: #1 write the polynomial in standard form, #2-4 factor and solve.

1. $x = -5, x = 3i$
 $(x+5)(x^2+9)$
 $x^3 + 5x^2 + 9x + 45$

$(x)^2 = (3i)^2$
 $x^2 = 9i^2$
 $x^2 = -9$
 $x^2 + 9 = 0$

2. $4x^2 + 21x - 18 = 0$

$4x^2 + 24x \mid -3x - 18$
 $4x(x+6) \mid -3(x+6)$
 $(4x-3)(x+6) = 0$
 $x = \frac{3}{4} \quad x = -6$

$\begin{array}{r} -72 \\ 1 \overline{) 72} \\ 2 \ 36 \\ \underline{-3 \ 24} \\ 4 \ 18 \\ 6 \ 12 \\ \underline{8 \ 9} \end{array}$

3. $x^3 + x^2 - 3x - 3 = 0$
 $x^2(x+1) \mid -3(x+1) = 0$
 $(x^2-3)(x+1) = 0$
 $x = \pm\sqrt{3} \quad x = -1$

4. $6x^3 - 25x^2 + 21x + 10 = 0$
 $x^2(6x-25) \mid +1(21x+10)$
 Not Factorable

Divide (old school):

1. $\begin{array}{r} 42,894 \\ 3 \overline{) 42894} \\ \underline{-3} \downarrow \\ 12 \\ \underline{-12} \downarrow \\ 08 \\ \underline{-6} \downarrow \\ 29 \\ \underline{-27} \downarrow \\ 24 \end{array}$

14,298

2. $\begin{array}{r} 22,345,623 \\ 63 \overline{) 22345623} \\ \underline{-189} \downarrow \\ 344 \\ \underline{-315} \downarrow \\ 295 \\ \underline{-252} \downarrow \\ 436 \\ \underline{-378} \downarrow \\ 582 \\ \underline{-567} \downarrow \\ 153 \\ \underline{-126} \\ 27 \end{array}$

$354,692 \frac{3}{7}$

How do we solve polynomial equations?

• $(2x-3)(2x+1)(x-5) = 0$

**What about from the warm-up?

$2x-3=0$ $2x+1=0$ $x-5=0$
 $x=\frac{3}{2}$ $x=-\frac{1}{2}$ $x=5$

$6x^3 - 25x^2 + 21x + 10 = 0$

Two options: Long Division OR Tabular Division

$$\frac{6x^3 - 25x^2 + 21x + 10}{x - 2}$$

Long division

$$\begin{array}{r}
 6x^2 - 13x - 5 \\
 x-2 \overline{) 6x^3 - 25x^2 + 21x + 10} \\
 \underline{- 6x^3 - 12x^2} \\
 -13x^2 + 21x \\
 \underline{- 13x^2 + 26x} \\
 -5x + 10 \\
 \underline{- 5x + 10} \\
 0
 \end{array}$$

Tabular division

	$6x^2$	$-13x$	-5	Remainder
x	$6x^3$	$-13x^2$	$-5x$	none
-2	$-12x^2$	$26x$	$+10$	

Remainder Theorem:

to check if your remainder is correct
 evaluate the expression using the root (divisor)

example : $x-2=0$ $6(2)^3 - 25(2)^2 + 21(2) + 10 = 0$
 (from above) $x=2$ \uparrow
 the remainder

L.D.

Examples:

1. $\frac{x^3-4x^2+2x-5}{x-3}$

$$\begin{array}{r} x^2 - 1x - 1 + \frac{-8}{x-3} \\ x-3 \overline{) x^3 - 4x^2 + 2x - 5} \\ \underline{-x^3 + 3x^2} \\ -1x^2 + 2x \\ \underline{-1x^2 + 3x} \\ -1x - 5 \\ \underline{-1x + 3} \\ -8 \end{array}$$

$x^2 - 1x - 1 + \frac{-8}{x-3}$

2. $\frac{2x^3-5x^2+8x-3}{2x-1}$

$$\begin{array}{r} x^2 - 2x + 3 \\ 2x-1 \overline{) 2x^3 - 5x^2 + 8x - 3} \\ \underline{-2x^3 + 1x^2} \\ -4x^2 + 8x \\ \underline{-4x^2 + 2x} \\ 6x - 3 \\ \underline{6x - 3} \\ 0 \end{array}$$

$x^2 - 2x + 3$

What is different about these?: missing a term

3. $\frac{x^3+x-2}{x+2}$

$$\begin{array}{r} x^2 - 2x + 5 + \frac{-12}{x+2} \\ x+2 \overline{) x^3 + 0x^2 + 1x - 2} \\ \underline{-x^3 + 2x^2} \\ -2x^2 + 1x \\ \underline{-2x^2 - 4x} \\ 5x - 2 \\ \underline{5x + 10} \\ -12 \end{array}$$

$x^2 - 2x + 5 + \frac{-12}{x+2}$

4. $\frac{x^4-2x^2+x-3}{x^2+x-1}$

$$\begin{array}{r} x^2 - 1x + 0 + \frac{-3}{x^2+x-1} \\ x^2+x-1 \overline{) x^4 + 0x^3 - 2x^2 + x - 3} \\ \underline{-x^4 + 1x^3 - 1x^2} \\ -1x^3 - 1x^2 + 1x \\ \underline{-1x^3 - 1x^2 + 1x} \\ 0x^2 + 0x - 3 \\ \underline{0x^2 + 0x + 0} \\ -3 \end{array}$$

$x^2 - 1x + \frac{-3}{x^2+x-1}$

5. $\frac{x^2+8}{x+2}$

$$\begin{array}{r} x-2 + \frac{12}{x+2} \\ x+2 \overline{) x^2+0x+8} \\ \underline{x^2+2x} \\ -2x+8 \\ \underline{-2x-4} \\ 12 \end{array}$$

$$x-2 + \frac{12}{x+2}$$

6. $\frac{18x^3+3x^2-61x+10}{3x-5}$

$$\begin{array}{r} 6x^2+11x-2 \\ 3x-5 \overline{) 18x^3+3x^2-61x+10} \\ \underline{18x^3-30x^2} \\ 33x^2-61x \\ \underline{-33x^2+55x} \\ -6x+10 \\ \underline{-6x+10} \\ 0 \end{array}$$

$$6x^2+11x-2$$

7. $\frac{2x^5-5x^4-9x^2-7x-5}{x^2+2}$

$$\begin{array}{r} 2x^3-5x^2-4x+1 + \frac{x-7}{x^2+2} \\ x^2+0x+2 \overline{) 2x^5-5x^4+0x^3-9x^2-7x-5} \\ \underline{2x^5+0x^4+4x^3} \\ -5x^4-4x^3-9x^2 \\ \underline{-5x^4+0x^3-10x^2} \\ -4x^3+1x^2-7x \\ \underline{-4x^3+0x^2-8x} \\ 1x^2+1x-5 \\ \underline{1x^2+0x+2} \\ 1x-7 \end{array}$$

$$2x^3-5x^2-4x+1 + \frac{x-7}{x^2+2}$$

8. Division (3 equivalent problems)

$$2x^3 + 7x^2 - 18x - 22 \div x + 4$$

$$2x^3 + 7x^2 - 18x - 22 * (x + 4)^{-1}$$

$$\frac{2x^3 + 7x^2 - 18x - 22}{x + 4}$$

$$\begin{array}{r} 2x^2-1x-14 + \frac{34}{x+4} \\ x+4 \overline{) 2x^3+7x^2-18x-22} \\ \underline{2x^3+8x^2} \\ -1x^2-18x-22 \\ \underline{-1x^2-4x} \\ -14x-22 \\ \underline{-14x-56} \\ 34 \end{array}$$

$$2x^2-1x-14 + \frac{34}{x+4}$$

T.P.

Examples:

1. $\frac{x^3 - 4x^2 + 2x - 5}{x - 3}$

	x^2	$-1x$	-1	R
x	x^3	$-1x^2$	$-1x$	-8
-3	$-3x^2$	$3x$	3	

$$x^2 - 1x - 1 + \frac{-8}{x-3}$$

2. $\frac{2x^3 - 5x^2 + 8x - 3}{2x - 1}$

	x^2	$-2x$	3	R
$2x$	$2x^3$	$-4x^2$	$6x$	0
-1	$-1x^2$	$2x$	-3	

$$x^2 - 2x + 3$$

What is different about these?:

missing a term

3. $\frac{x^3 + x - 2}{x + 2}$

	x^2	$-2x$	5	R
x	x^3	$-2x^2$	$5x$	-12
2	$2x^2$	$-4x$	10	

$$x^2 - 2x + 5 + \frac{-12}{x+2}$$

4. $\frac{x^4 - 2x^2 + x - 3}{x^2 + x - 1}$

	x^2	$-x$	0	R
x^2	x^4	$-x^3$	$0x^2$	$0x$
x	x^3	$-x^2$	$0x$	-3
-1	$-1x^2$	$1x$	0	

$$x^2 - x + \frac{-3}{x^2 + x - 1}$$

5. $\frac{x^2+8}{x+2}$

	x	-2	R
X	x ²	-2x	12
2	2x	-4	

$$x - 2 + \frac{12}{x+2}$$

6. $\frac{18x^3+3x^2-61x+10}{3x-5}$

	6x ²	11x	-2	R
3x	18x ³	33x ²	-6x	0
-5	-30x ²	-55x	10	

$$6x^2 + 11x - 2$$

7. $\frac{2x^5-5x^4-9x^2-7x-5}{x^2+2}$

	2x ³	-5x ²	-4x	1	
X ²	2x ⁵	-5x ⁴	-4x ³	1x ²	1x
0x	0x ⁴	0x ³	0x ²	0x	-7
2	4x ³	-10x ²	-8x	2	

$$2x^3 - 5x^2 - 4x + 1 + \frac{x-7}{x^2+2}$$

8. Division (3 equivalent problems)

$$2x^3 + 7x^2 - 18x - 22 \div x + 4$$

$$2x^3 + 7x^2 - 18x - 22 * (x + 4)^{-1}$$

$$\frac{2x^3 + 7x^2 - 18x - 22}{x + 4}$$

	2x ²	-1x	-14	R
X	2x ³	-1x ²	-14x	34
4	8x ²	-4x	-56	

$$2x^2 - 1x - 14 + \frac{34}{x+4}$$