

Rational Expressions

1. Which of the following are rational numbers, and which are not?

$\frac{3}{4}, 3.14, \pi, \frac{5}{0}, -\sqrt{17}, 23, \frac{1+\sqrt{5}}{2}, -1, 6.022 \times 10^{23}, 0, 4i$

<u>Rational Numbers</u>
$\frac{3}{4}, 3.14, 0, -1, 23, 6.022 \times 10^{23}$
<u>Not rational</u>
$\pi, \frac{5}{0}, \sqrt{17}, \frac{1+\sqrt{5}}{2}, 4i$

Definition of rational: Expression that can be written as a fraction $\frac{P}{Q}$
 $P \neq 0$ are polynomials and $Q \neq 0$

Doesn't mean irrational

Steps for simplifying:

- ① Factor numerator
- ② Factor denominator
- ③ Restrictions
- ④ Simplify (cancel)

What are restrictions? Values of x that will make the denominator 0

How do we find them? Solve for x in denominator

Examples: Find an equivalent rational expression in lowest terms (don't forget to state restrictions).

1. $\frac{16n}{20n} \quad n \neq 0 \quad \left(\frac{4}{5} \right)$

2. $\frac{(x+3)(x-2)}{(x-2)(2x+5)} \quad x \neq 2, -\frac{5}{2}$
 $\left(\frac{x+3}{2x+5} \right)$

3. $\frac{(3+x)(6x-7)}{(x+3)(6x+7)} \quad x \neq -3, -\frac{7}{6}$
 $\left(\frac{6x-7}{6x+7} \right)$

4. $\frac{4x-2}{6-12x} \quad \frac{2(2x-1)}{6(1-2x)} \quad x \neq \frac{1}{2}$
 $\left(-\frac{1}{3} \right)$

5. $\frac{5x^2+13x-6}{x^2-9x-36}$
 $\frac{(x+3)(5x-2)}{(x-12)(x+3)}$
 $x^2+15x-2x-6$
 $5x(x+3)-2(x+3)$
 $x \neq 12, -3 \quad \left(\frac{5x-2}{x-12} \right)$

6. $\frac{x^2-16}{3x^2+10x-8}$
 $\frac{(x+4)(x-4)}{(x+4)(3x-2)}$
 $3x^2+12x-2x-8$
 $3x(x+4)-2(x+4)$
 $x \neq -4, \frac{2}{3}$
 $\left(\frac{x-4}{3x-2} \right)$

Rule for multiplying rational expressions:

If $a, b, c,$ and d are integers with $c \neq 0$ and $d \neq 0$, then

$$\frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}$$

Steps for multiplying:

- ① Factor all numerators
- ② Factor all denominators

* Think of as 1 giant fraction

$$\frac{\text{numerators}}{\text{denominators}}$$

- ③ Restrictions
- ④ Simplify (cancel)
- ⑤ Multiply numerators or denominators if necessary

Examples:

a. $\left(\frac{3x-6}{2x+6}\right) \cdot \left(\frac{5x+15}{6x+12}\right)$

$$\frac{3(x-2) \cancel{5(x+3)}}{2(\cancel{x+3}) 6(x+2)}$$

$x \neq -3$
 $x \neq -2$

$$\frac{15(x-2)}{12(x+2)} = \frac{5(x-2)}{4(x+2)}$$

b. $\frac{x^2+3x-28}{8x-72} \cdot \frac{x^2-16x+63}{x^2-49}$

$$\frac{\cancel{(x+7)}(x-4) \cancel{(x-7)}(x-9)}{8(x-9) \cancel{(x+7)}(x-7)}$$

$x \neq 9, -7, 7$

$$\frac{(x-4)}{8}$$

You try:

Find the following products and reduce to lowest terms, state restrictions:

1. $\left(\frac{2x+6}{x^2+x-6}\right) \cdot \left(\frac{x^2-4}{2x}\right)$

$$\frac{\cancel{2(x+3)} \cancel{(x+2)}(x-2)}{(\cancel{x+3})(x-2) 2x}$$

$x \neq -3, 2, 0$

$$\frac{(x+2)}{x}$$

2. $\left(\frac{x^2-3x}{x^2-10x+21}\right) \cdot \left(\frac{x^2-12x+35}{5x^2-25x}\right)$

$$\frac{x \cancel{(x-3)} \cancel{(x-7)}(x-5)}{(\cancel{x-7})(\cancel{x-3}) 5x(x-5)}$$

$x \neq 7, 3, 0, 5$

$$\frac{1}{5}$$

How do we divide fractions - rational expressions :

Recall that division is the same as multiplying by the inverse.

Ex. $15 \div \frac{1}{3}$ is the same as $15 * 3 = 45$

Ex2. $\frac{3}{5} \div \frac{6}{7}$ is the same as $\frac{3}{5} * \frac{7}{6} = \frac{21}{30} = \frac{7}{10}$

Rule for dividing rational expressions:

If $a, b, c,$ and d are integers with $b \neq 0, c \neq 0,$ and $d \neq 0,$ then

$$\frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \cdot \frac{d}{b}$$

Examples:

a. $\frac{x-3}{x^2-7x+6} \div \frac{x^2-x-6}{x-1}$

$x \neq 6, 1, 1$
 $3, -2$

$$\frac{(x-3)}{(x-6)(x-1)} \div \frac{(x-3)(x+2)}{(x-1)}$$

$$\frac{(x-3)}{(x-6)(x-1)} \cdot \frac{(x-1)}{(x-3)(x+2)}$$

$$\frac{1}{(x-6)(x+2)}$$

b. $\frac{x^2-2x-24}{x^2-4} \div \frac{x^2+3x-4}{x^2+x-2}$

$x \neq -2, 2, 1$
 -4

$$\frac{(x-6)(x+4)}{(x+2)(x-2)} \div \frac{(x+4)(x-1)}{(x+2)(x-1)}$$

$$\frac{(x-6)(x+4)}{(x+2)(x-2)} \cdot \frac{(x+2)(x-1)}{(x+4)(x-1)}$$

$$\frac{(x-6)}{(x-2)}$$

Steps for dividing:

- 1) Factor numerator
- 2) Factor denominator
- 3) Restrictions
- 4) Change to multiplication
(flip 2nd fraction)
- 5) Restrictions AGAIN
- 6) Simplify (cancel)
- 7) Multiply across if necessary

You try:

Find the following quotients and reduce to lowest terms, state restrictions:

1. $\left(\frac{16x-24}{4x^2-9}\right) \div \left(\frac{12x+36}{x^2-8x-33}\right)$

$x \neq -\frac{3}{2}, \frac{3}{2}, 11, -3$

$\frac{8(2x-3)}{(2x+3)(2x-3)} = \frac{12(x+3)}{(x-11)(x+3)}$

$\frac{8(2x-3)(x-11)(x+3)}{(2x+3)(2x-3)12(x+3)}$

$\frac{8(x-11)}{12(2x+3)} = \frac{2(x-11)}{3(2x+3)}$

2. $\frac{\left(\frac{x^2-15x+54}{25x^2-4}\right)}{\left(\frac{x^2-81}{5x^2-2x}\right)}$

$x \neq \frac{2}{5}, \frac{2}{5}, 0, -1, 9$

$\frac{(x-6)(x-9)}{(5x+2)(5x-2)} \div \frac{(x+9)(x-9)}{x(5x-2)}$

$\frac{(x-6)(x-9)x(5x-2)}{(5x+2)(5x-2)(x+9)(x-9)}$

$\frac{x(x-6)}{(5x+2)(x+9)}$

Review and Preview:

1. $\frac{x^2-16}{5x^2+16x-16}$

$\frac{5x^2+20x-4x-16}{5x(x+4)-4(x+4)}$

$\frac{(x+4)(x-4)}{(x+4)(5x-4)}$

$\frac{(x-4)}{(5x-4)}$

$x \neq -4, \frac{4}{5}$

2. $\frac{x^2+6x-16}{x^2+3x-40} * \frac{x^2+6x-55}{x^2+9x-22}$

$x \neq -8, 5, -11, 0$

$\frac{(x+8)(x-2)(x+11)(x-5)}{(x+8)(x-5)(x+11)(x-2)}$

1

3. $\frac{18x-108}{x^2+9x+8} \div \frac{36x^2-36x}{x^2-1}$

$x \neq -8, -1, 1, 0$

$\frac{18(x-6)}{(x+8)(x+1)} \div \frac{36x(x-1)}{(x+1)(x-1)}$

$\frac{18(x-6)}{(x+8)(x+1)} \cdot \frac{(x+1)(x-1)}{36x(x-1)}$

$\frac{18(x-6)}{36x(x+8)} = \frac{(x-6)}{2x(x+8)}$

4. $\frac{x^2+9x-80}{x^2-36} - \frac{3x-8}{x^2-36}$

$\frac{x^2+6x-72}{x^2-36}$

$\frac{(x+12)(x-6)}{(x+6)(x-6)}$

$x \neq -6, 6$

$\frac{(x+12)}{(x+6)}$