Alg2 Unit 3 (3.1) – Reference Angles and Intro to Unit Circle Name: Key

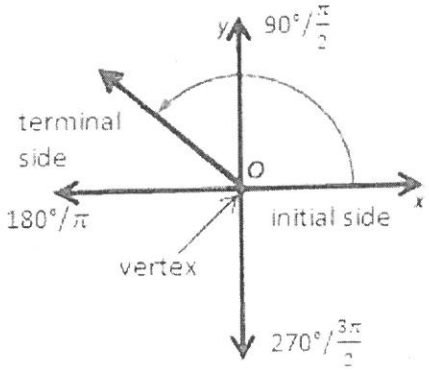
****What do we do when angles are greater than 180 degrees?****

important Vocabulary Terms

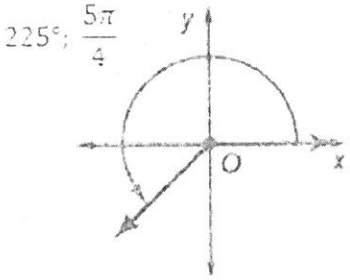
Initial side: the ray fixed on the positive x-axis

Terminal side: a ray starting at the origin, where the angle ends

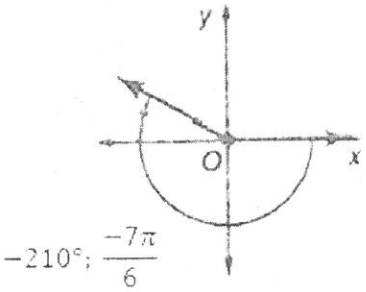
Standard Position: angle whose vertex is at the origin w/ inital side at positive x-axis



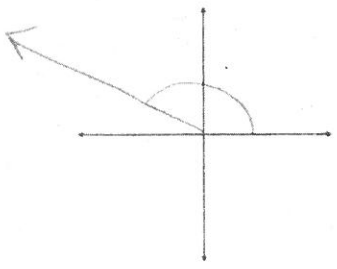
Positive Angle Measure counterclockwise



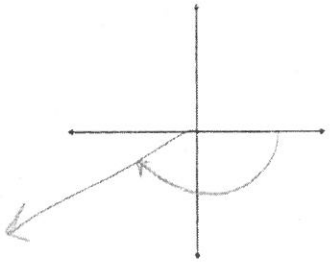
Negative Angle Measure clockwise



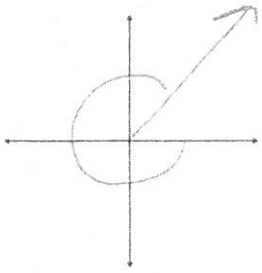
Ex.1 Draw a 150° angle in standard position.



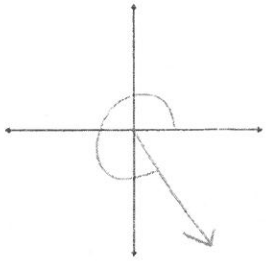
Ex. 2 Draw a $(-\frac{5\pi}{6})$ angle in standard position.



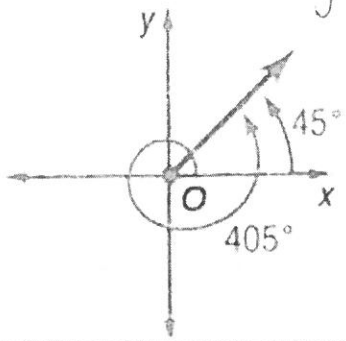
Ex. 3 Draw $(-315°)$ in standard position.



Ex. 4 Draw $\frac{5\pi}{3}$ in standard position.



• **Coterminal Angles** – 2 angles in standard position that have the same terminal side



**Key Concept

To find a positive measure angle:
$+ 360^\circ$ or 2π

To find a negative measure angle:
$- 360$ or 2π

Ex. Find one angle with a positive measure and one angle with a negative measure that is coterminal with a $\frac{4\pi}{3}$ angle.

$\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$ $\quad$ $\frac{4\pi}{3} - \frac{6\pi}{3} = -\frac{2\pi}{3}$

Ex. Find one angle with a positive measure and one angle with a negative measure that is coterminal with a 100° angle.

$100^\circ + 360^\circ = 460^\circ$ $\quad$ $100^\circ - 360^\circ = -260^\circ$

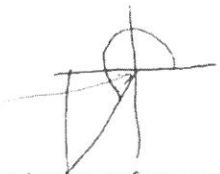
Ex. Find one angle with a positive measure and one angle with a negative measure that is coterminal with a $\frac{7\pi}{9}$ angle. $\frac{7\pi}{9} + \frac{18\pi}{9} = \frac{25\pi}{9}$ $\quad$ $\frac{7\pi}{9} - \frac{18\pi}{9} = -\frac{11\pi}{9}$

Ex. Find one angle with a positive measure and one angle with a negative measure that is coterminal with a -90° angle. $-90^\circ + 360^\circ = 270^\circ$ $\quad$ $-90^\circ - 360^\circ = -450^\circ$

• **Reference Angle** θ' : the acute angle formed by the terminal side of the angle and the x-axis

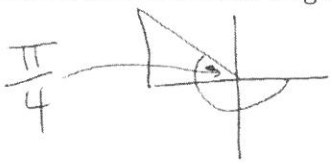
Ex 1. What is the reference angle for 240°?

60°

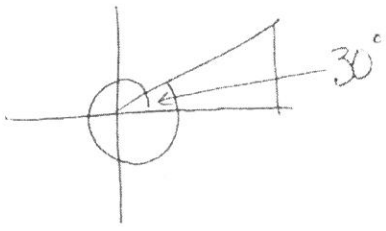


Ex 2. What is the reference angle for $-\frac{5\pi}{4}$?

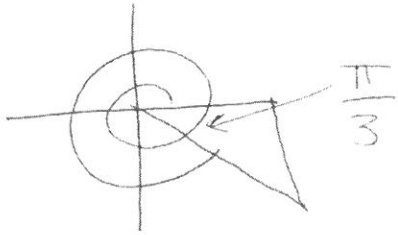
$\frac{\pi}{4}$



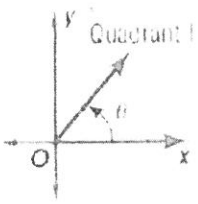
Ex 3. What is the reference angle 390°?

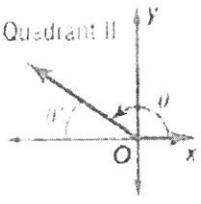


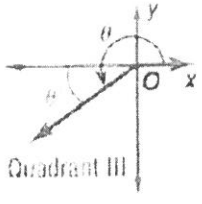
Ex 4. What is the reference angle for $\frac{11\pi}{3}$? $3\frac{2}{3}\pi$

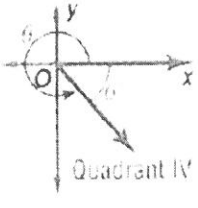


**Key Concept



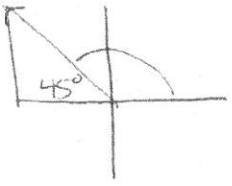




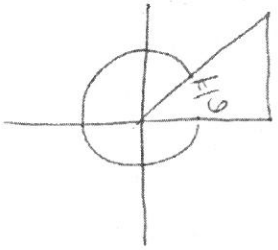


You Try. Find the reference angle.

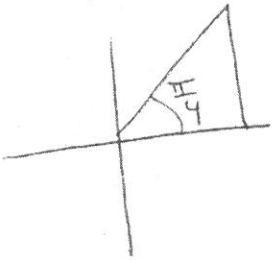
1. 135°



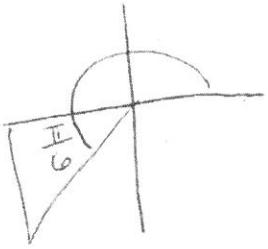
2. $-\frac{11\pi}{6}$



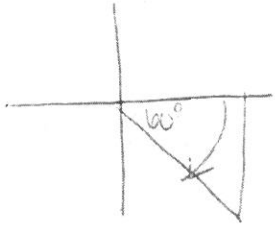
3. $\frac{\pi}{4}$



4. $\frac{7\pi}{6}$

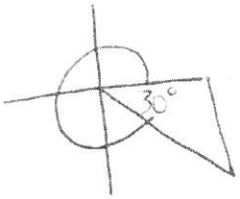


5. -60°

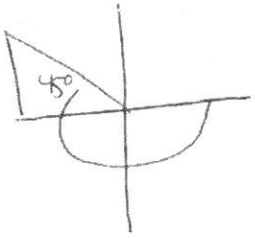


6. 690°

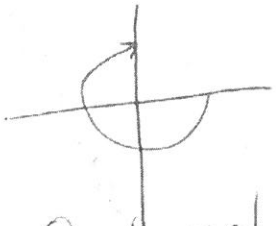
690
−360
330



7. -225°



8. $-\frac{3\pi}{2}$



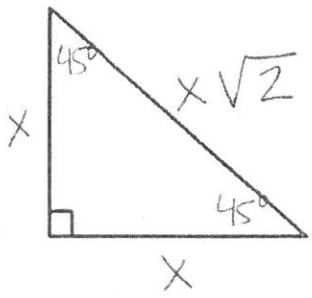
Quadrantal
No reference angle

Prior Knowledge:

Special Right Triangles

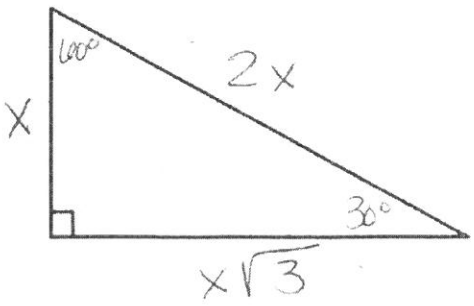
$45-45-90$

$\frac{\pi}{4}-\frac{\pi}{4}-\frac{\pi}{2}$



$30-60-90$

$\frac{\pi}{6}-\frac{\pi}{3}-\frac{\pi}{2}$



Recall:

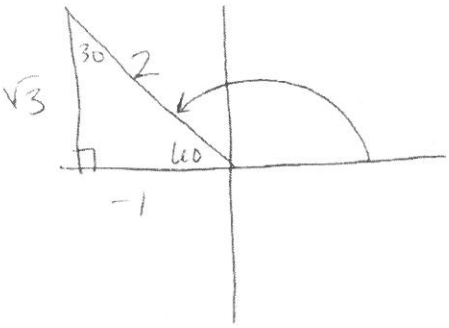
1. $\sin 30° = \frac{1}{2}$
2. $\cos 60° = \frac{1}{2}$
3. $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$
4. $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$
5. $\tan 45° = 1$
6. $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

****Key Concept: How do you find the exact value and correct sign of the trig function?**

1. Find the reference angle.
2. Find the value of the Trig Function.
3. Assign the proper sign to your value from step 2.

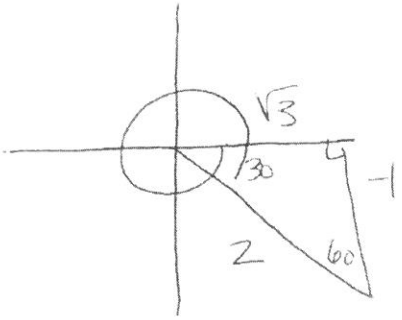
Hypot. always positive

Ex: Find: $\sin 120°$



$\frac{O}{H}$ $\frac{\sqrt{3}}{2}$

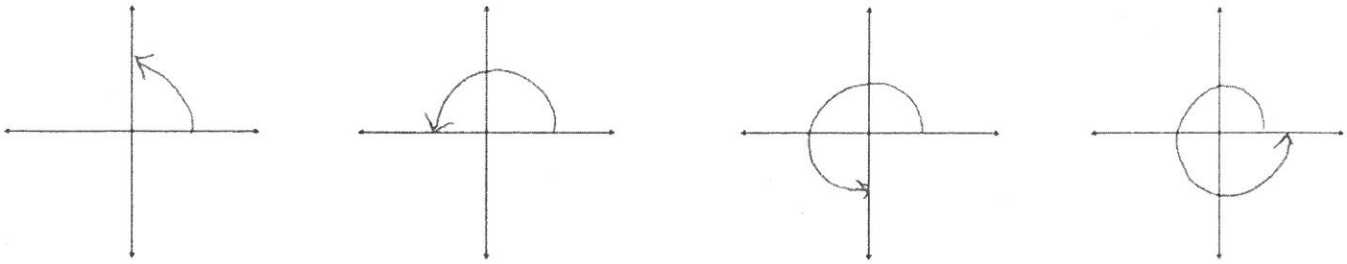
Ex: Find: $\tan \frac{-13\pi}{6}$ $-390°$



$\frac{O}{A}$ $\frac{-1}{\sqrt{3}}$

• **Quadrantal Angles** – angles that have their terminal side on an axis

Draw each of the positive quadrantal angles on the coordinate planes below (use degrees and radians).



$\theta = 90°$ $\quad x = \frac{\pi}{2}$

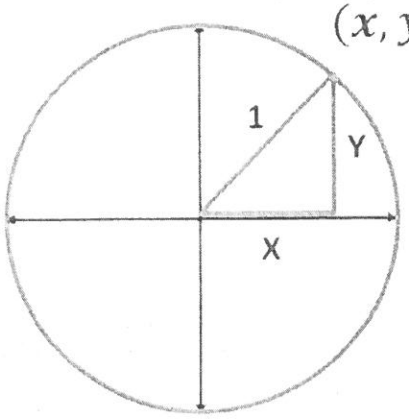
$\theta = 180°$ $\quad x = \pi$

$\theta = 270°$ $\quad x = \frac{3\pi}{2}$

$\theta = 0°$ or $360°$ $\quad x = 0\pi$ or 2π

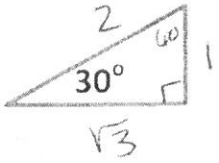
How to find the exact value of a Quadrantal

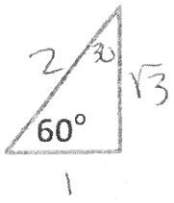
$(\cos\theta, \sin\theta)$
(x, y)

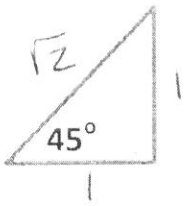


In a unit circle, the radius is one.

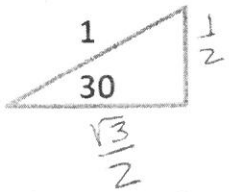
Reminder of our special right triangle ratios.

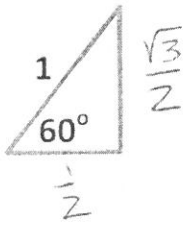


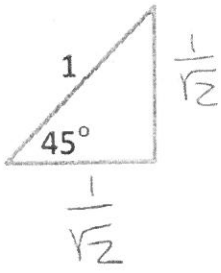




How would they look if you changed the hypotenuse to 1?







Look at the horizontal (x) values? Do they look familiar? $\cos\theta = \frac{A}{H}$

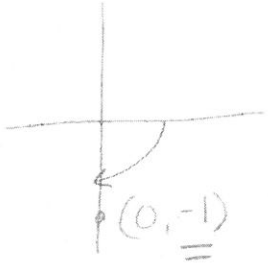
What about the vertical (y) values? $\sin\theta = \frac{O}{H}$

How do you find $tan\theta$ (what is the ratio)? $\frac{\sin}{\cos}$

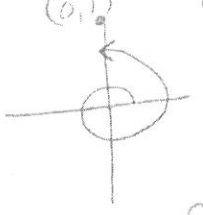
How can you relate that to what you just found about x and y? $(\cos\theta, \sin\theta)$

You try: Find

$\sin\left(-\frac{\pi}{2}\right) = -1$



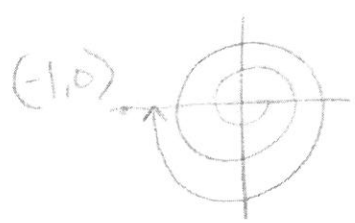
$\sec 450°$ cos $\frac{450}{-360}$ $\frac{}{90}$



$\cos \to \frac{0}{1}$

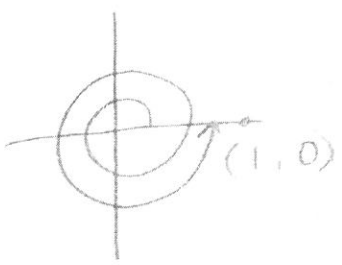
$\sec \to \frac{1}{0}$ undefined

$\tan(-5\pi)$



$\frac{0}{-1} = 0$

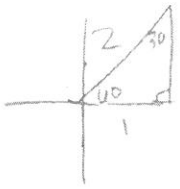
$\csc 720°$ sin



$\frac{0}{1} \to \frac{1}{0}$ undefined

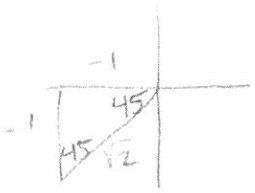
You Try. Find the exact value (no calculator!!!)

1. $\cos(-300°) = \frac{1}{2}$



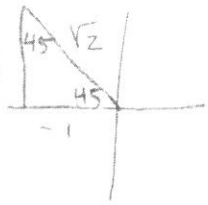
$\frac{A}{H} = \frac{1}{2}$

2. $\tan\frac{5\pi}{4} = 1$



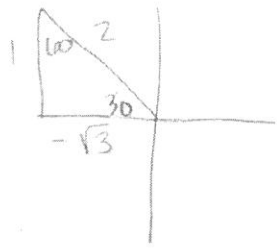
$\frac{O}{A} = \frac{-1}{-1}$

3. $\sin\frac{11\pi}{4} = \frac{1}{\sqrt{2}}$



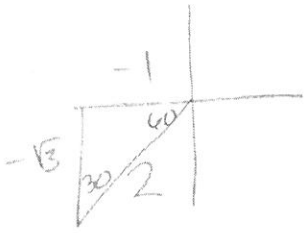
$\frac{O}{H} = \frac{1}{\sqrt{2}}$

4. $\tan(-210°) = \frac{1}{-\sqrt{3}}$



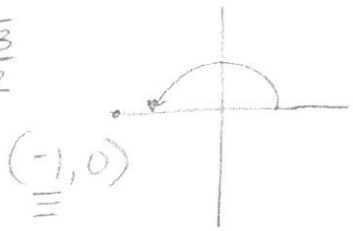
$\frac{O}{A} = \frac{1}{-\sqrt{3}}$

5. $\sin(-\frac{2\pi}{3}) = \frac{-\sqrt{3}}{2}$

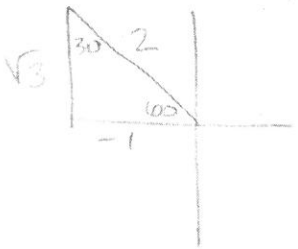


$\frac{O}{H} = \frac{-\sqrt{3}}{2}$

6. $\cos 180° = -1$

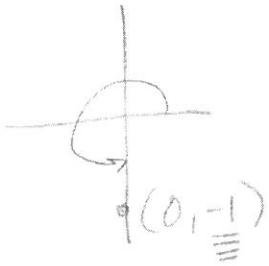


7. $\tan 120^0$ $\frac{\sqrt{3}}{-1}$ or $-\sqrt{3}$

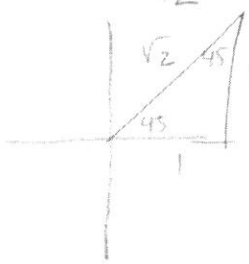


$\frac{O}{A} = \frac{\sqrt{3}}{-1}$

8. $\sin 270^0$ -1

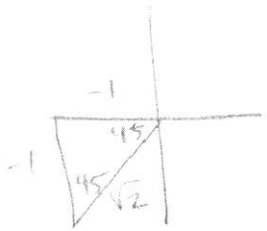


9. $\sin(\frac{\pi}{4})$ $\frac{1}{\sqrt{2}}$



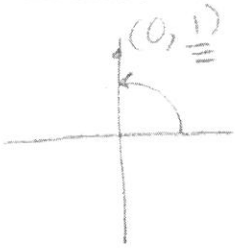
$\frac{O}{H} = \frac{1}{\sqrt{2}}$

10. $\tan 225^0$ 1

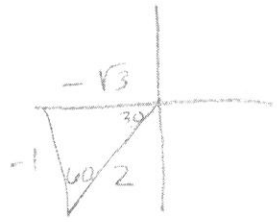


$\frac{O}{A} = \frac{-1}{-1}$

11. $\sin 90^0$ 1

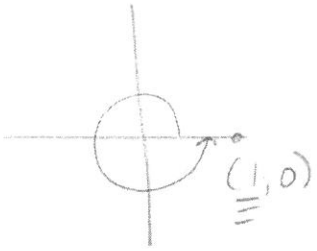


12. $\cos(\frac{7\pi}{6})$ $\frac{-\sqrt{3}}{2}$

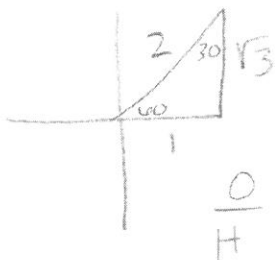


$\frac{A}{H} = \frac{-\sqrt{3}}{2}$

13. $\cos 360^0$ 1

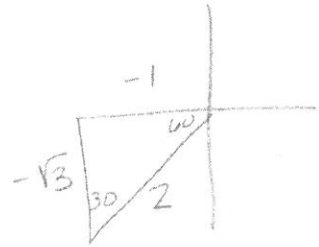


14. $\sin(\frac{\pi}{3})$ $\frac{\sqrt{3}}{2}$



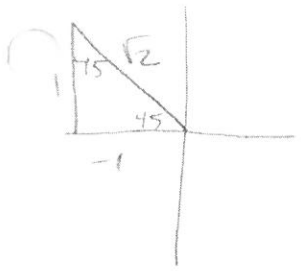
$\frac{O}{H} = \frac{\sqrt{3}}{2}$

15. $\tan(-120^0)$ $\sqrt{3}$



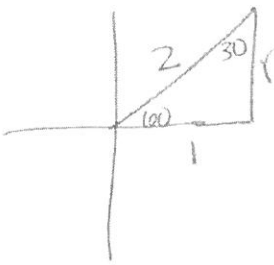
$\frac{O}{A} = \frac{-\sqrt{3}}{-1}$

16. $\tan(\frac{3\pi}{4})$ -1



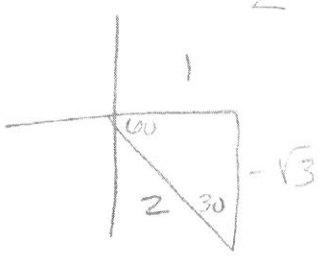
$\frac{O}{A} = \frac{1}{-1}$

17. $\cos(\frac{\pi}{3})$ $\frac{1}{2}$



$\frac{A}{H} = \frac{1}{2}$

18. $\sin 300^0$ $-\frac{\sqrt{3}}{2}$



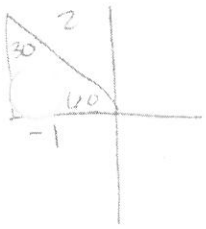
$\frac{O}{H} = \frac{-\sqrt{3}}{2}$

Key Concept: Let's meet the other trigonometric ratios that we haven't discussed yet!

$$csc\theta = \frac{hyp}{opp} \qquad sec\theta = \frac{hyp}{adj} \qquad cot\theta = \frac{adj}{opp}$$

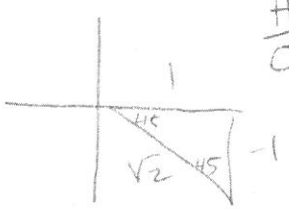
Find the exact values of the following:

1. $\cot 120°$ $-\frac{1}{\sqrt{3}}$



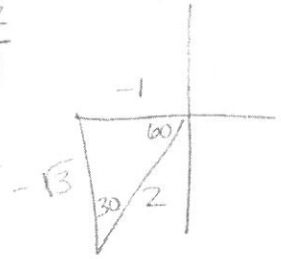
$\frac{A}{O} = \frac{-1}{\sqrt{3}}$

2. $\csc(-\frac{\pi}{4})$ $-\sqrt{2}$



$\frac{H}{O} = \frac{\sqrt{2}}{-1}$

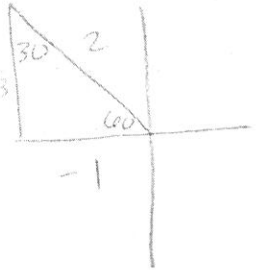
3. $\sec(-\frac{8\pi}{3})$ -2



$\frac{H}{A} = \frac{2}{-1}$

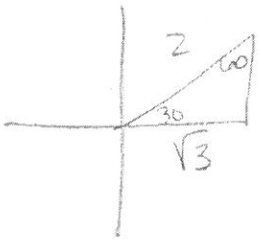
$\tan = \frac{sin}{cos} = \frac{0}{-1}$

4. $\csc(\frac{2\pi}{3})$ $\frac{2}{\sqrt{3}}$



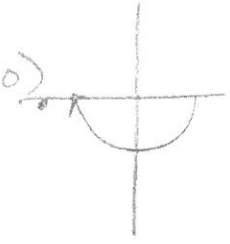
$\frac{H}{O} = \frac{2}{\sqrt{3}}$

5. $\sec(-330^0)$ $\frac{2}{\sqrt{3}}$



$\frac{H}{A} = \frac{2}{\sqrt{3}}$

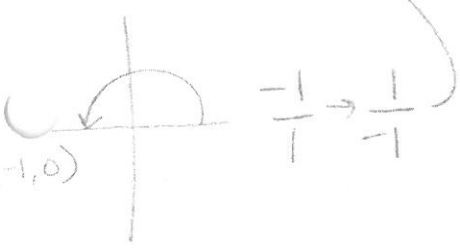
6. $\cot -\pi$ undefined



$\frac{-1}{0} =$

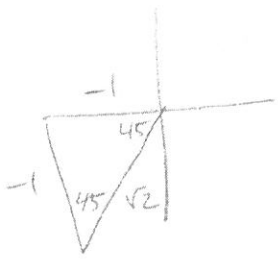
$\cos -1$

7. $\sec 180^0$ -1



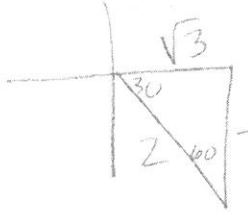
$\frac{-1}{1} \to \frac{1}{-1}$

8. $\cot 225^0$ 1



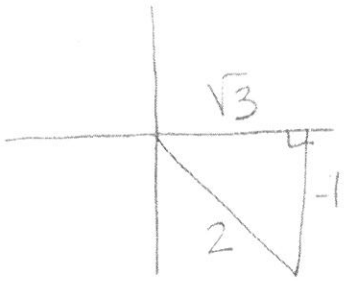
$\frac{A}{O} = \frac{-1}{-1}$

9. $\csc(\frac{11\pi}{6})$ -2



$\frac{H}{O} = \frac{2}{-1}$

Ex: In Quadrant IV the $cos\theta = \frac{\sqrt{3}}{2}$. Find the exact values of the other five trig ratios.



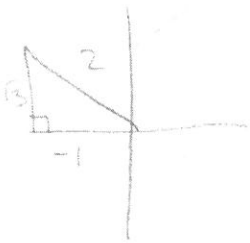
$\sin\theta = -\frac{1}{2}$ $\quad \csc\theta = -2$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad \sec\theta = \frac{2}{\sqrt{3}}$

$\tan\theta = \frac{-1}{\sqrt{3}}$ $\quad \cot\theta = -\sqrt{3}$

You Try. Find the exact values of the other 5 trig ratios.

1. In Quadrant II the $cos\theta = -\frac{1}{2}$.

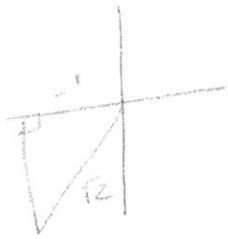


$\sin\theta = \frac{\sqrt{3}}{2}$ $\quad \csc\theta = \frac{2}{\sqrt{3}}$

$\cos\theta = \frac{-1}{2}$ $\quad \sec\theta = \frac{2}{-1}$

$\tan\theta = -\sqrt{3}$ $\quad \cot\theta = \frac{1}{-\sqrt{3}}$

2. In Quadrant III the $sin\theta = -\frac{1}{\sqrt{2}}$.

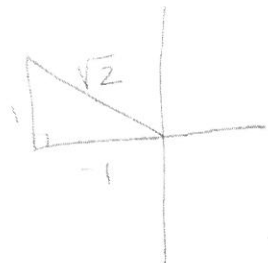


$\sin\theta = \frac{-1}{\sqrt{2}}$ $\quad \csc\theta = -\sqrt{2}$

$\cos\theta = \frac{-1}{\sqrt{2}}$ $\quad \sec\theta = -\sqrt{2}$

$\tan\theta = 1$ $\quad \cot\theta = 1$

3. In Quadrant II the $tan\theta = -1$.

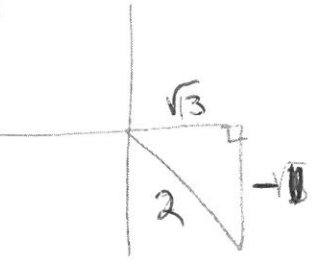


$\sin\theta = \frac{1}{\sqrt{2}}$ $\quad \csc\theta = \frac{\sqrt{2}}{1}$

$\cos\theta = \frac{-1}{\sqrt{2}}$ $\quad \sec\theta = \frac{\sqrt{2}}{-1}$

$\tan\theta = -1$ $\quad \cot\theta = -1$

4. In Quadrant IV the $cot\theta = -\frac{\sqrt{3}}{1}$.



$\sin\theta = \frac{-1}{2}$ $\quad \csc\theta = \frac{-2}{1}$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad \sec\theta = \frac{2}{\sqrt{3}}$

$\tan\theta = \frac{-1}{\sqrt{3}}$ $\quad \cot\theta = \frac{\sqrt{3}}{-1}$