

Key

**Compositions:** the range of the first function becomes the domain of the second function...a series of substitutions

Recall:

$$\text{if } f(x) = x^2 - 2x + 7$$

$$\text{then } f(5) = (5)^2 - 2(5) + 7$$

$$\text{so } f(5) = 22$$

In other words: if the input is 5 the output is 22; (5, 22)

To do a composition you will do substitution TWICE

$$g(x) = 3x - 4 \text{ and } f(x) = x^2 - 2x + 7$$

$$g(f(5)) \quad \leftarrow \text{do } f(5) \text{ first (see above), since } f(5) = 22 \\ \text{substitute 22 in the place of } f(5)$$

$$g(22) \rightarrow 3(22) - 4 \quad \leftarrow \text{do } g(22) \text{ second}$$

$$g(22) = 62$$

Try  $f(g(5))$

More Examples:  $f(x) = -8x + 2$

$$g(x) = 2x^2 - 4$$

$$h(x) = \frac{5x-2}{4}$$

1.  $f(h(6)) \quad \frac{5(6)-2}{4} \rightarrow 7$

$$-8(7) + 2 \\ -54$$

2.  $h(f(6))$

$$-8(6) + 2 \rightarrow -46 \\ \frac{5(-46) - 2}{4} \\ -58$$

3.  $g(f(-2))$

$$-8(-2) + 2 \rightarrow 18$$

$$2(18)^2 - 4 \\ 644$$

4.  $f(g(-2))$

$$2(-2)^2 - 4 \rightarrow 4$$

$$-8(4) + 2 \\ -30$$

Compositions of Functions: keep in mind it is just substitution

$$f(x) = 4x$$

$$g(x) = 2x - 4$$

$$h(x) = x^2 + 1 \quad j(x) = \frac{1}{2}x + 2$$

1.  $f(g(x))$

$$4(2x-4)$$
$$8x-16$$

2.  $g(f(x))$

$$2(4x)-4$$
$$8x-4$$

3.  $g(h(x))$

$$2(x^2+1)-4$$
$$2x^2+2-4$$

4.  $h(g(x))$

$$(2x-4)^2+1$$
$$(2x-4)(2x-4)+1$$
$$4x^2-16x+16+1$$
$$4x^2-16x+17$$

5.  $g(j(x))$

$$2\left(\frac{1}{2}x+2\right)-4$$
$$x+4-4$$
$$x$$

6.  $j(g(x))$

$$\frac{1}{2}(2x-4)+2$$
$$x-2+2$$
$$x$$

7.  $g(g(x))$

$$2(2x-4)-4$$
$$4x-8-4$$
$$4x-12$$