Unit 2 (1.3) **Exponents and Logarithms Notes** Key

Warm-Up:

1. $64 = 8^{x-6}$

$8^2 = 8^{x-6}$

$2 = x - 6$

$8 = x$

2. $100^x = 100{,}000$

$10^{2x} = 10^5$

$2x = 5$

$x = \frac{5}{2}$

3. $9^{-x+3} = 27^{x-3}$

$3^{2(-x+3)} = 3^{3(x-3)}$

$-2x + 6 = 3x - 9$

$-5x = -15$

$x = 3$

4. $16^{2x} = 9^{3x-5}$

Need a new method

Evaluate each expression. The first two have been completed for you.

a. $\text{WhatPower}_2(8) = 3$ $\quad 2^3 = 8$	b. $\text{WhatPower}_3(9) = 2$ $\quad 3^2 = 9$
c. $\text{WhatPower}_6(36) =$ 2 $\quad 6^{?} = 36$	d. $\text{WhatPower}_2(32) =$ 5 $\quad 2^{?} = 32$
e. $\text{WhatPower}_{10}(1000) =$ 3 $\quad 10^{?} = 1000$	f. $\text{WhatPower}_{10}(1{,}000{,}000) =$ 6 $\quad 10^{?} = 1{,}000{,}000$
g. $\text{WhatPower}_{100}(1{,}000{,}000) =$ 3 $\quad 100^{?} = 1{,}000{,}000$	h. $\text{WhatPower}_4(64) =$ 3 $\quad 4^{?} = 64$
i. $\text{WhatPower}_2(64) =$ 6 $\quad 2^{?} = 64$	j. $\text{WhatPower}_9(3) =$ $\frac{1}{2}$ $\quad 9^{?} = 3$
k. $\text{WhatPower}_5(\sqrt{5}) =$ $\frac{1}{2}$ $\quad 5^{?} = \sqrt{5}$	l. $\text{WhatPower}_{\frac{1}{2}}\left(\frac{1}{8}\right) =$ 3 $\quad \left(\frac{1}{2}\right)^{?} = \frac{1}{8}$
m. $\text{WhatPower}_{42}(1) =$ 0 $\quad 42^{?} = 1$	

•Logarithms help get the variable out of the exponent. Logs are inverses of exponents.

$y = b^x$ $(exponential\ form)$ **is equivalent to:** $log_b(y) = x$ $(logarithmic\ form)$

Example: $2^3 = 8$ **is equivalent to** $log_2 8 = 3$

Convert the following into exponential form:

1. $log_5 15625 = 6$

$5^6 = 15,625$

2. $log_{\frac{1}{2}}\left(\frac{1}{16}\right) = 4$

$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

3. $log_3 27 = 3$

$3^3 = 27$

Convert the following into logarithmic form:

4. $7^3 = 343$

$\log_7 343 = 3$

5. $\frac{1}{2}^{-5} = 32$

$\log_{\frac{1}{2}} 32 = -5$

6. $4^{-3} = \frac{1}{64}$

$\log_4\left(\frac{1}{64}\right) = -3$

What is the purpose of being able to change from one form to the other? get the variable away from the log or out of the exponent

$log_3 x = -5$

$10^x = 100,000$

Practice:

Put the following in exponential form and solve.

1. $log_2 64 = x$

$2^x = 64$

$2^x = 2^6$

$x = 6$

2. $log_3 81 = x$

$3^x = 81$

$3^x = 3^4$

$x = 4$

3. $log_4\left(\frac{1}{16}\right) = x$

$4^x = \left(\frac{1}{16}\right)$

$4^x = 4^{-2}$

$x = -2$

4. $log_5 x = -2$

$5^{-2} = x$

$\frac{1}{25} = x$

5. $log_{49} x = \frac{1}{2}$

$49^{\frac{1}{2}} = x$

$7 = x$

6. $log_{16} 64 = x$

$16^x = 64$

$4^{2x} = 4^3$

$2x = 3$

$x = \frac{3}{2}$

7. $log_4(5x + 1) = 2$

$4^2 = 5x+1$

$16 = 5x-1$

$15 = 5x$

$3 = x$

8. $log_6(9x) = 3$

$6^3 = 9x$

$216 = 9x$

$24 = x$

9. $log_2(2x - 4) = 3$

$2^3 = 2x-4$

$8 = 2x-4$

$12 = 2x$

$6 = x$

Evaluate.

- $\log_9(729) =$ 3
- $\log_3\left(\frac{1}{243}\right) =$ -5
- $\log_4(4096) =$ 6
- $\log_2(-16) =$ X
- $\log_{121}(11) =$ $\frac{1}{2}$ (square root)
- $\log_{512}(8) =$ $\frac{1}{3}$ (cube root)
- $\log_{37}(1) =$ 0
- $\log_{+5}(+78125) =$ 7

Warm-up:

Solve for x.

1. $log_4(2x-4)=3$

$4^3=2x-4$

$64=2x-4$

$68=2x$

$x=34$

2. $4^{x+5}=16^x$

$4^{x+5}=4^{2x}$

$x+5=2x$

$5=x$

Properties of Logarithms: *These properties only work if the bases are the same.*	
Property of Equality $\boldsymbol{log_b x = log_b y}$ (if $b>0,\ b\neq 1$ then $x=y$) Example: $log_7 x = log_7 3$ $x=3$	Product Property $\boldsymbol{log_b mn = log_b m + log_b n}$ (if m, n, b are positive, $b\neq 1$) Example: $log_7 12 + log_7 3 = log_7(12\cdot 3)$ $log_7 36$
Quotient Property $\boldsymbol{log_b\left(\frac{m}{n}\right) = log_b m - log_b n}$ (if m, n, b are positive, $b\neq 1$) Example: $log_7 12 - log_7 3 = log_7(\frac{12}{3})$ $log_7 4$	Power Property $\boldsymbol{log_b m^p = p log_b m}$ (if m, b are positive, $b\neq 1$ and p is a real number) Examples: $log_7 2^3 = 3log_7 2$ $2log_7 6 = log_7 36$

Practice:

1. $log_6 4x + log_6 3 = log_6 84$

$\log_6(4x\cdot 3)=\log_6 84$

$12x=84$

$x=7$

2. $log_2(x+2) - log_2(x-4) = 2$

$\log_2\frac{(x+2)}{(x-4)}=2$

$2^2=\frac{x+2}{x-4}$

$4(x-4)=x+2$

$4x-16=x+2$

$3x=18$

$x=6$

3. $log_4 x + log_4 8 = 4$

$\log_4 8x = 4$

$4^4 = 8x$

$256 = 8x$

$32 = x$

4. $log_{11} 6 + 3log_{11} 2 = log_{11} 3x$

$\log_{11} 6 + \log_{11} 2^3 = \log_{11} 3x$

$48 = 3x$

$16 = x$

5. $5log_9 2 = log_9 4x$

$\log_9 2^5 = \log_9 4x$

$32 = 4x$

$8 = x$

6. $3log_5 4 - log_5 2 = log_5 16x$

$\log_5 \frac{4^3}{2} = \log_5 16x$

$32 = 16x$

$2 = x$

7. $log_6 4x - log_6 3 = 2$

$\log_6 \frac{4x}{3} = 2$

$6^2 = \frac{4x}{3}$

$108 = 4x$

$27 = x$

8. $4log_{14} 3 = 2log_{14} x$

$\log_{14} 3^4 = \log_{14} x^2$

$81 = x^2$

$9 = x$

9. $log_2 5 + log_2 4x = 7$

$\log_2 20x = 7$

$2^7 = 20x$

$128 = 20x$

$6.4 = x$

10. $log_5 x + log_5(x+4) = log_5 32$

$\log_5 (x^2+4x) = \log_5 32$

$x^2+4x-32=0$

$(x+8)(x-4)=0$

$x = \cancel{-8},\ x = 4$

11. $log_{12} 6x - log_{12}(x+3) = log_{12} 4$

$\log_{12} \frac{6x}{x+3} = \log_{12} 4$

$\frac{6x}{x+3} = 4$

$4x+12 = 6x$

$12 = 2x$

$6 = x$

12. $3log_3 3 = log_3(4x-11)$

$\log_3 3^3 = \log_3 4x-11$

$27 = 4x-11$

$38 = 4x$

$9.5 = x$

13. $6log_4 2 - log_4\left(\frac{1}{2}x+1\right) = 4log_4 2$

$\log_4 \frac{2^6}{\frac{1}{2}x+1} = \log_4 2^4$

$\frac{64}{\frac{1}{2}x+1} = 16$

$8x+16 = 64$

$8x = 48$

$x = 6$

14. $2log_6 12 + log_6 x = 4$

$\log_6 12^2 + \log_6 x = 4$

$\log_6 144x = 4$

$144x = 6^4$

$144x = 1296$

$x = 9$