Unit 2 (2.4) Inverses Notes

Key

## Finding Inverses:

**The domain of a relation becomes the range of the inverse, and the range of a relation becomes the domain of the inverse.**

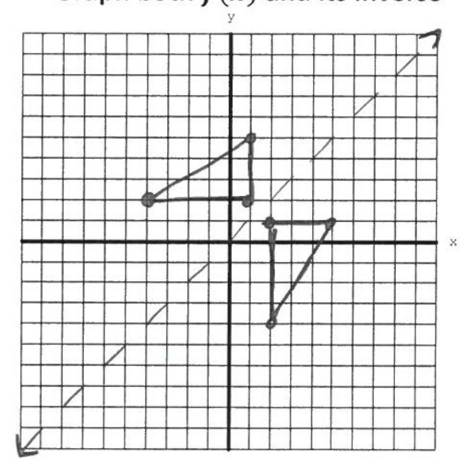
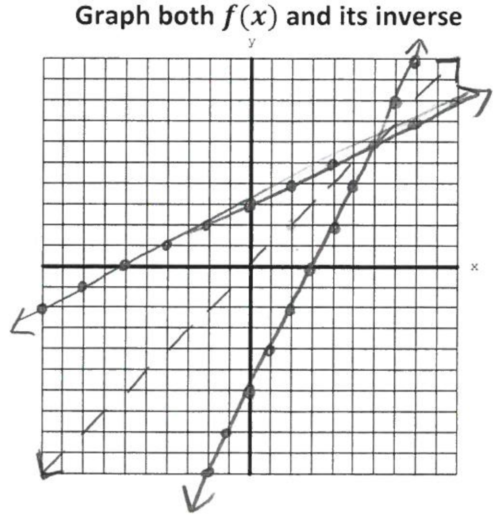
To put it more simply: switch x & y

The graph of the original relation will be reflected over $y = x$

Steps:

1. Replace $f(x)$ with y.
2. Switch x and y.
3. Solve for y.
4. Replace y with $f^{-1}(x)$.

Examples:

| **Find the Inverse** | **Graph both $f(x)$ and its inverse** |
| --- | --- |
| 1. $(2,1), (5,1), (2,-4)$ (coordinates of the vertices of a right triangle)<br>$(1,2)\ (1,5)\ (-4,2)$ |  |
| **Find the Inverse** | **Graph both $f(x)$ and its inverse** |
| 2. $f(x) = 2x - 6$<br>$y = 2x - 6$<br>$x = 2y - 6$<br>$x + 6 = 2y$<br>$\frac{1}{2}x + 3 = y$ |  |

**Find the Inverse**

3. $f(x) = \frac{2x+12}{6} \rightarrow y = \frac{1}{3}x + 2$

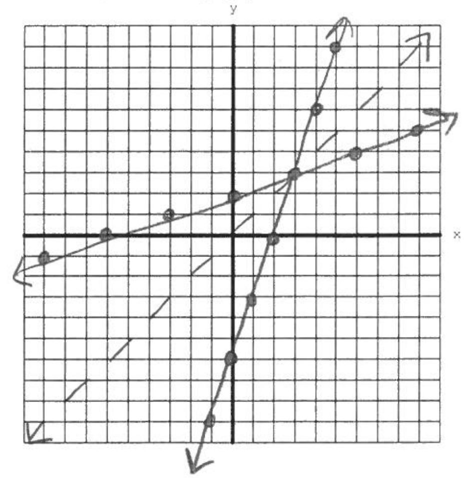
$x = \frac{2y+12}{6}$

$6x = 2y + 12$

$6x - 12 = 2y$

$3x - 6 = y$

**Graph both $f(x)$ and its inverse**



**Find the Inverse**

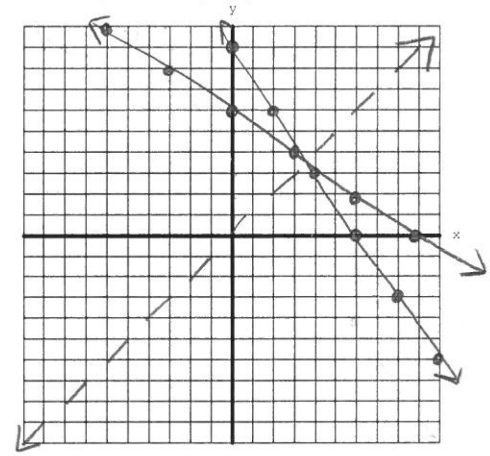
4. $f(x) = -\frac{3}{2}x + 9$

$x = -\frac{3}{2}y + 9$

$-\frac{2}{3}(x-9) = -\frac{3}{2}y \cdot -\frac{2}{3}$

$-\frac{2}{3}x + 6 = y$

**Graph both $f(x)$ and its inverse**



## Finding the inverse of a non-linear equation:

1. Find $b^{-1}(x) : b(x) = \sqrt{2x-4}$

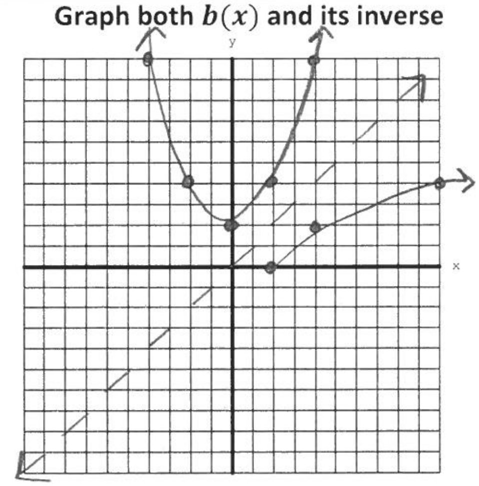
$x = \sqrt{2y-4}$

$x^2 = 2y - 4$

$x^2 + 4 = 2y$

$\frac{x^2+4}{2} = y$

**Graph both $b(x)$ and its inverse**



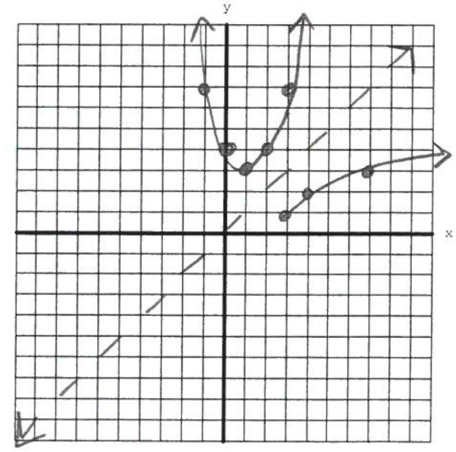
2. Find $e^{-1}(x)$: $e(x) = \sqrt{x-3} + 1$

$x = \sqrt{y-3} + 1$

$x - 1 = \sqrt{y-3}$

$(x-1)^2 + 3 = y$

**Graph both $e(x)$ and its inverse**



3. Find $c^{-1}(x)$: $c(x) = (x+4)^2 - 2$

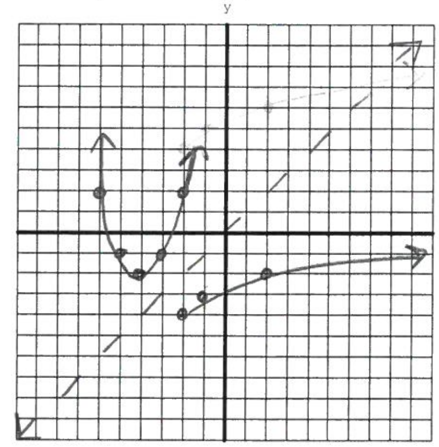
$x = (y+4)^2 - 2$

$x + 2 = (y+4)^2$

$\sqrt{x+2} = y + 4$

$\sqrt{x+2} - 4 = y$

**Graph both $c(x)$ and its inverse**



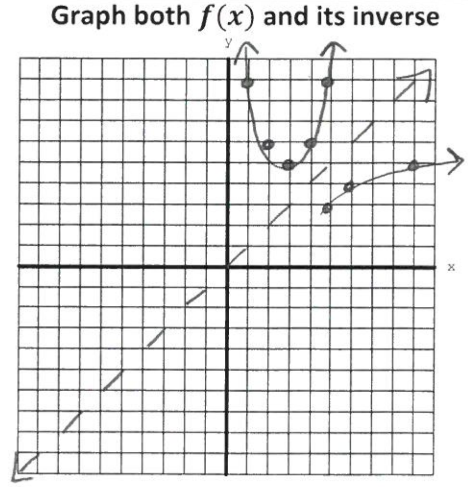
4. Find $f^{-1}(x)$ : $f(x) = (x-3)^2 + 5$

$x = (y-3)^2 + 5$

$x - 5 = (y-3)^2$

$\sqrt{x-5} + 3 = y$

**Graph both $f(x)$ and its inverse**



## If you don't have a graph, how do you know you solved for the inverse correctly?

**Proving $f(x)$ and $f^{-1}(x)$ are inverses.**

Check to see if the compositions of $f(x)$ and $f^{-1}(x)$ is the identity function. ($y = x$)

$$f\left(f^{-1}(x)\right) = x \qquad \& \qquad f^{-1}(f(x)) = x$$

PROVE that the following are inverses by using compositions.

1. $f(x) = \frac{3}{4}x - 6$ and $g(x) = \frac{4}{3}x + 8$

$\frac{3}{4}\left(\frac{4}{3}x+8\right)-6$ $\qquad$ $\frac{4}{3}\left(\frac{3}{4}x-6\right)+8$

$x+6-6$ $\qquad$ $x-8+8$

$x$ $\qquad$ $x$

2. $f(x) = 2x^2 - 1$ and $g(x) = \sqrt{\frac{x+1}{2}}$

$2\left(\sqrt{\frac{x+1}{2}}\right)^2-1$ $\qquad$ $\sqrt{\frac{2x^2-1+1}{2}}$

$2\left(\frac{x+1}{2}\right)-1$ $\qquad$ $\sqrt{\frac{2x^2}{2}}$

$x+1-1$ $\qquad$ $\sqrt{x^2}$

$x$ $\qquad$ $x$

3. $f(x) = \frac{1}{3}x + 10$ and $g(x) = 3x - 30$

$\frac{1}{3}(3x-30)+10$ $\qquad$ $3\left(\frac{1}{3}x+10\right)-30$

$x-10+10$ $\qquad$ $x+30-30$

$x$ $\qquad$ $x$

4. $f(x) = \frac{x^2+3}{2}$ and $g(x) = \sqrt{2x-3}$

$\frac{\left(\sqrt{2x-3}\right)^2+3}{2}$ $\qquad$ $\sqrt{2\left(\frac{x^2+3}{2}\right)-3}$

$\frac{2x-3+3}{2}$ $\qquad$ $\sqrt{x^2+3-3}$

$\frac{2x}{2}$ $\qquad$ $\sqrt{x^2}$

$x$ $\qquad$ $x$