

Unit 3 (4.6) Horizontal Transformations of Trig. Functions

Key

Warm-up: Describe the transformation of each function.

1. $y = \frac{1}{2}x^2$

V.D. $\frac{1}{2}$

2. $y = \sqrt{x+6}$

H.T. -6

3. $y = |x-7| - 9$

H.T. 7

V.T. -9

4. $y = (x-2)^2 + 5$

H.T. 2

V.T. 5

5. $y = -3|x-1| + 7$

Reflection

H.T. 1

V.T. 7
V.D. 3

6. $y = \sqrt{x+5} - 10$

H.T. -5

V.T. -10

Objective: To understand how horizontal dilations and translations affect the graphs of the sine and cosine functions

$y = A \sin B(\theta - C) + D$

$y = A \cos B(\theta - C) + D$

"B" Horizontal Dilution:

Changes the period (1 cycle will fit in this space)

* H.D. is not listed in the equation

H.D. is the reciprocal of B

H.D. = $\frac{1}{B}$ $P = \frac{360 \text{ or } 2\pi}{B}$ $B = \frac{360 \text{ or } 2\pi}{P}$

"C" Horizontal Translation (phase shift/horizontal shift):

Shifts graph \leftarrow or \rightarrow (opposite)

moves the critical points

Describe the transformation for each of the following:

a. $y = \sin 3x$

$B = 3$ H.D. = $\frac{1}{3}$

$P = \frac{2\pi}{3}$

$\frac{2\pi}{3} + \frac{2\pi}{3} + \frac{2\pi}{3} = 2\pi$

This graph will be $\frac{1}{3}$ the size!

b. $y = \cos(\theta + 60^\circ)$

H.T. -60°

1st critical point will be -60° instead of 0°

c. $y = \cos \frac{x}{4}$ $y = \cos \frac{1}{4}x$

$B = \frac{1}{4}$ H.D. 4

$P = \frac{2\pi}{\frac{1}{4}} = 8\pi$ \leftarrow 4 times the size!

d. $y = \sin\left(x - \frac{\pi}{6}\right)$

H.T. $\frac{\pi}{6}$

e. $y = \cos[2(\theta - 90^\circ)]$

H.T. 90°

$B = 2$ H.D. = $\frac{1}{2}$

$P = \frac{360}{2} = 180^\circ$ \leftarrow $\frac{1}{2}$ the size

f. $y = \sin\left[\frac{1}{5}\left(x + \frac{\pi}{4}\right)\right]$

H.T. $-\frac{\pi}{4}$

$B = \frac{1}{5}$ H.D. 5

$P = \frac{2\pi}{\frac{1}{5}} = 10\pi$ 5 times the size!

Unit 3 (4.6) Horizontal Transformations of Trig. Functions

Write an equation for either the sine or cosine function with the given transformation(s):

a. h.d. of 2

$$y = \sin \frac{1}{2} \theta$$

b. h.t. of -50°

$$y = \cos(\theta + 50^\circ)$$

c. period of 45° and phase shift of 90°

$$B = \frac{360}{45} = 8$$

$$y = \cos[8(\theta - 90^\circ)]$$

d. horizontal shift of π

$$y = \sin(x + \pi)$$

e. period of $\frac{\pi}{3}$

$$B = \frac{2\pi}{\frac{\pi}{3}} = 6$$

$$y = \cos 6x$$

f. ~~h.d.~~ of ~~period~~ and h.t. of $\frac{\pi}{4}$

$$B = \frac{2\pi}{12\pi} = \frac{1}{6}$$

$$y = \sin \frac{1}{6} \left(x - \frac{\pi}{4}\right)$$

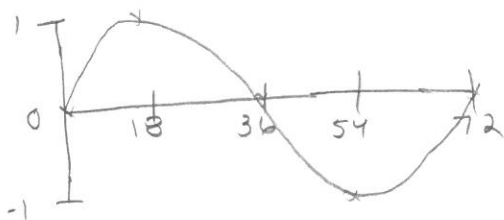
How to set up the x-axis:

Graph the functions.

1. $y = \sin 5\theta$

$$P = \frac{360}{5} = 72^\circ \quad \frac{72^\circ}{4} = 18^\circ$$

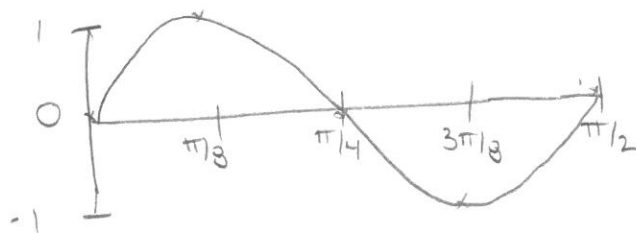
0, 18, 36, 54, 72



2. $y = \sin 4x$

$$P = \frac{2\pi}{4} = \frac{\pi}{2} \quad \frac{\pi}{2} \div 4 = \frac{\pi}{8}$$

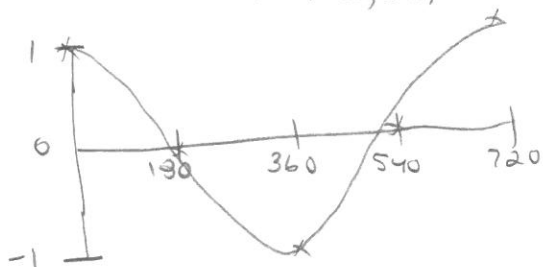
0, $\frac{4\pi}{8}$, $\frac{2\pi}{8}$, $\frac{3\pi}{8}$, $\frac{4\pi}{8}$



3. $y = \cos \frac{1}{2} \theta$

$$P = \frac{360}{\frac{1}{2}} = 720^\circ \quad \frac{720^\circ}{4} = 180^\circ$$

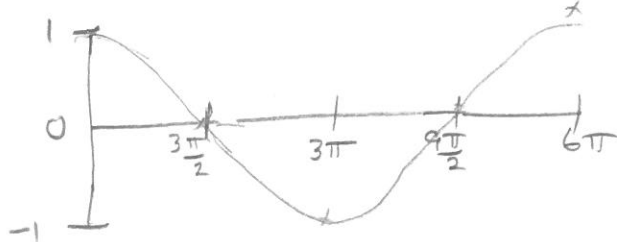
0, 180, 360, 540, 720



4. $y = \cos \frac{1}{3} x$

$$P = \frac{2\pi}{\frac{1}{3}} = 6\pi \quad \frac{6\pi}{4} = \frac{3\pi}{2}$$

0, $\frac{3\pi}{2}$, $\frac{6\pi}{2}$, $\frac{9\pi}{2}$, $\frac{12\pi}{2}$

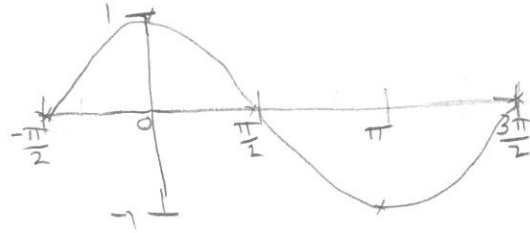
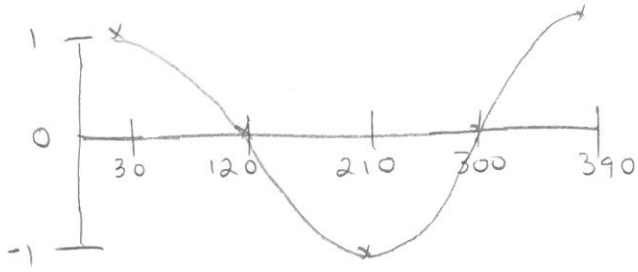


Unit 3 (4.6) Horizontal Transformations of Trig. Functions

$$P = \frac{2\pi}{1} = 2\pi \quad \frac{2\pi}{4} = \frac{\pi}{2}$$

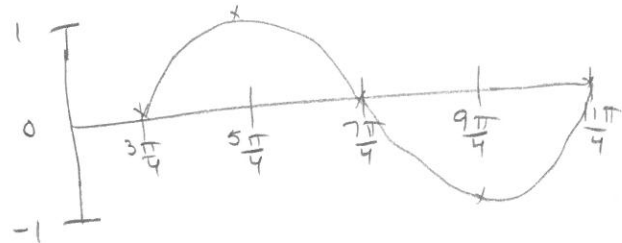
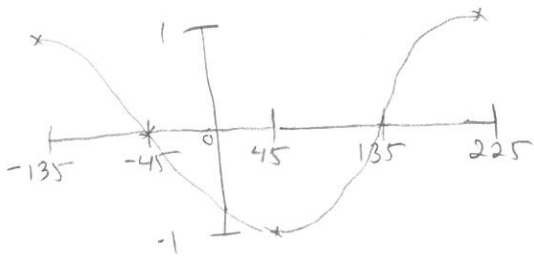
5. $y = \cos(\theta - 30^\circ)$ $P = \frac{360}{1} = 360^\circ$ $\frac{360}{4} = 90^\circ$ 6. $y = \sin\left(x + \frac{\pi}{2}\right)$

$$-\frac{\pi}{2}, 0, \frac{\pi}{2}, \frac{2\pi}{2}, \frac{3\pi}{2}$$



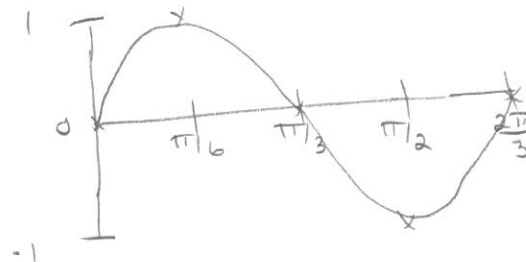
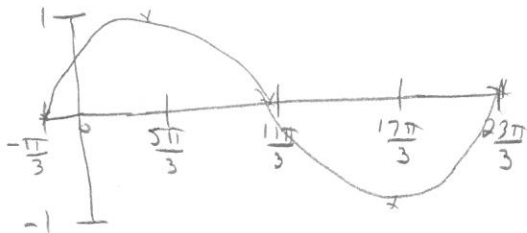
7. $y = \cos(\theta + 135^\circ)$ $P = \frac{360}{1} = 360^\circ$ $\frac{360}{4} = 90^\circ$
 $-135, -45, 45, 135, 225$

8. $y = \sin\left(x - \frac{3\pi}{4}\right)$ $P = \frac{2\pi}{1} = 2\pi$ $\frac{2\pi}{4} = \frac{\pi}{2}$ or $\frac{2\pi}{4}$
 $\frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$



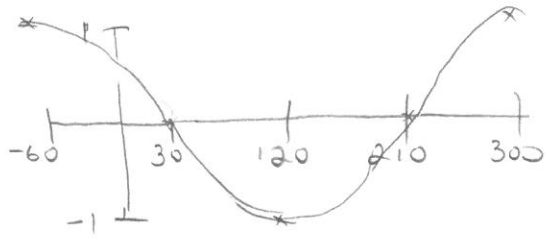
9. $y = \sin\frac{1}{4}\left(x + \frac{\pi}{3}\right)$ $P = \frac{2\pi}{\frac{1}{4}} = 8\pi$ $\frac{8\pi}{4} = 2\pi$ or $\frac{6\pi}{3}$
 $-\frac{\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{3}, \frac{17\pi}{3}, \frac{23\pi}{3}$

10. $y = \sin 3x$ $P = \frac{2\pi}{3}$ $\frac{2\pi}{3} \div 4 = \frac{2\pi}{12}$ or $\frac{\pi}{6}$
 $0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}$

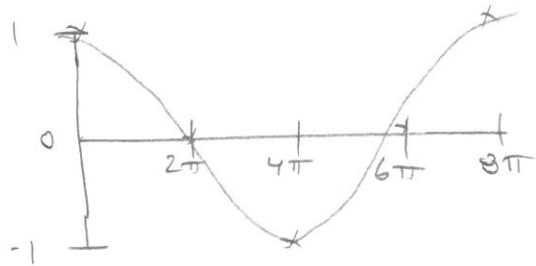


Unit 3 (4.6) Horizontal Transformations of Trig. Functions

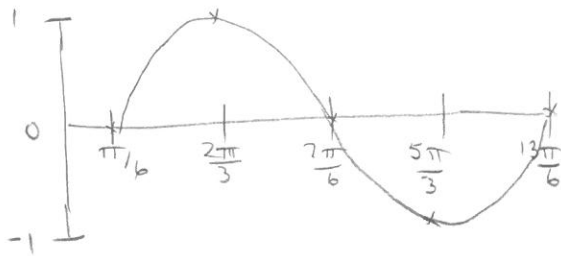
11. $y = \cos(\theta + 60^\circ)$ $P = \frac{360}{1} = 360^\circ$ $\frac{360}{4} = 90^\circ$
 $-60, 30, 120, 210, 300$



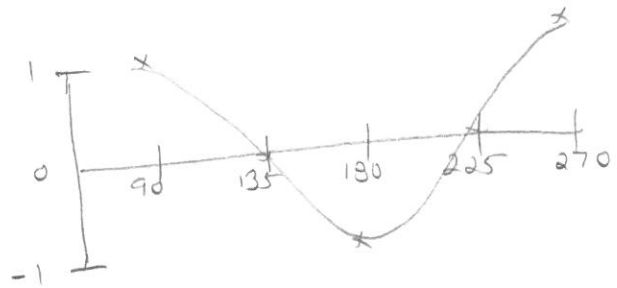
12. $y = \cos \frac{x}{4}$ $P = \frac{2\pi}{\frac{1}{4}} = 8\pi$ $\frac{8\pi}{4} = 2\pi$
 $0, 2\pi, 4\pi, 6\pi, 8\pi$
 $y = \cos \frac{1}{4}x$



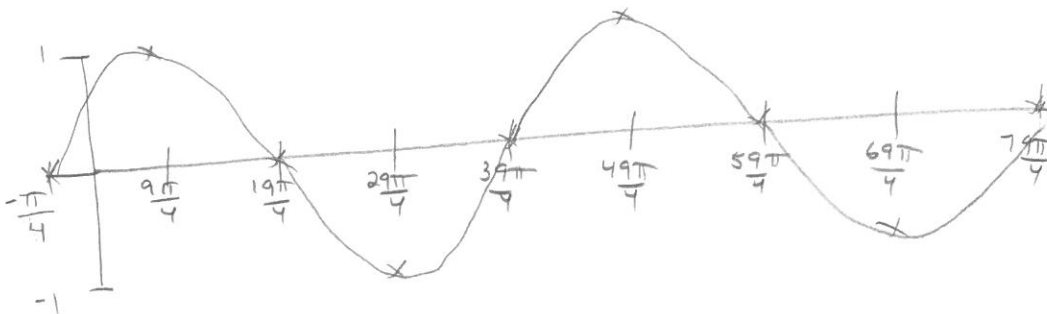
13. $y = \sin(x - \frac{\pi}{6})$ $P = \frac{2\pi}{1} = 2\pi$ $\frac{2\pi}{4} = \frac{\pi}{2}$ or $\frac{3\pi}{6}$
 $\frac{\pi}{6}, \frac{4\pi}{6}, \frac{7\pi}{6}, \frac{10\pi}{6}, \frac{13\pi}{6}$



14. $y = \cos 2(\theta - 90^\circ)$ $P = \frac{360}{2} = 180^\circ$ $\frac{180}{4} = 45^\circ$
 $90, 135, 180, 225, 270$

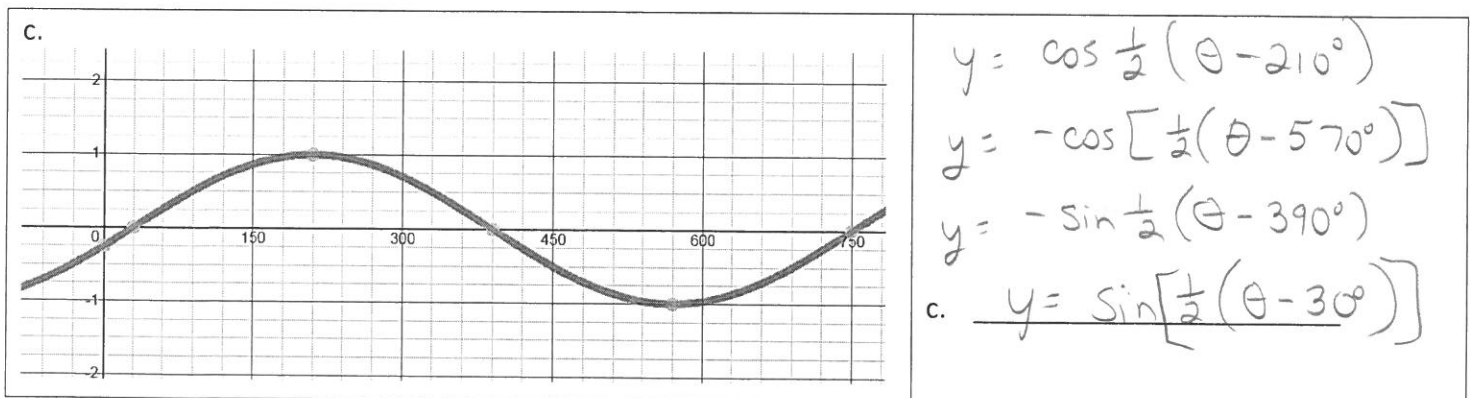
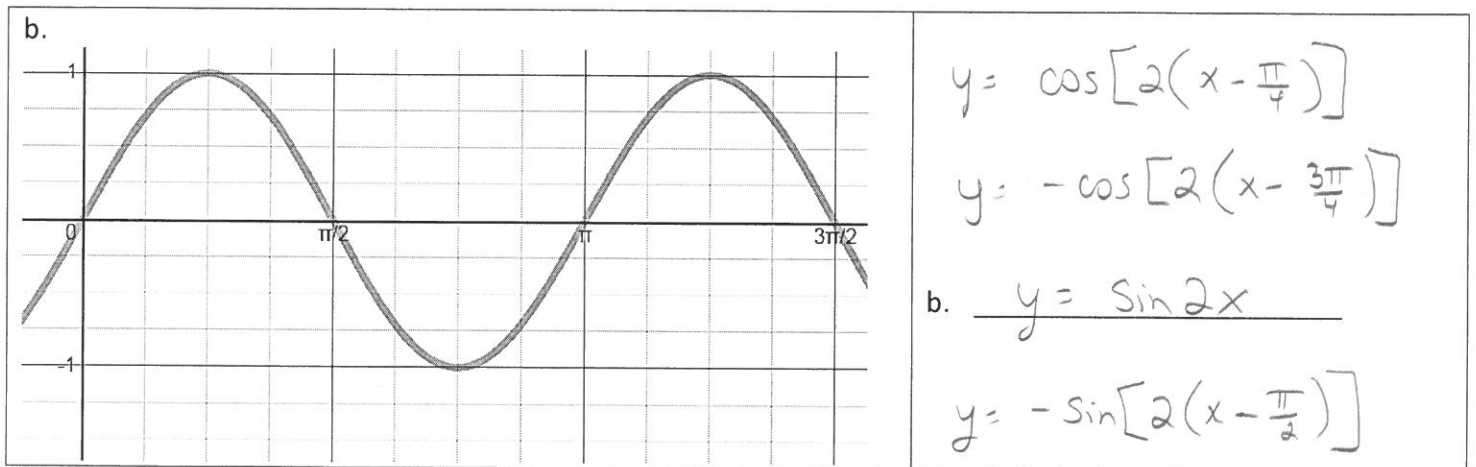
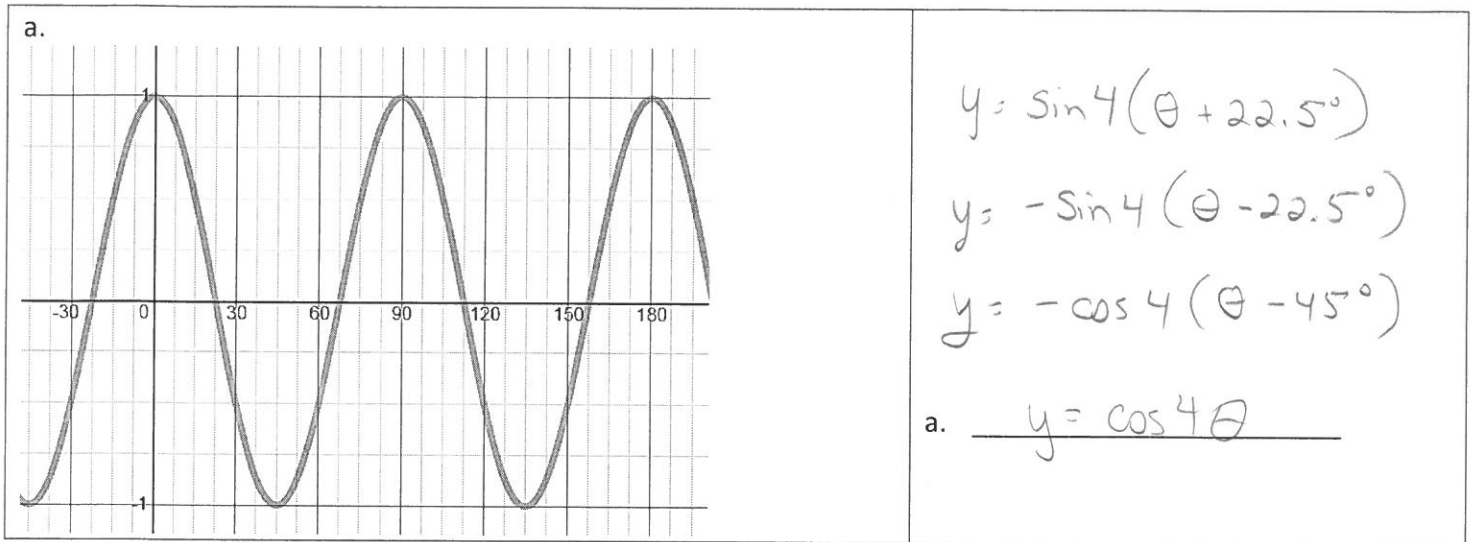


15. Sketch 2 cycles of $y = \sin\left[\frac{1}{5}\left(x + \frac{\pi}{4}\right)\right]$ $P = \frac{2\pi}{\frac{1}{5}} = 10\pi$ $\frac{10\pi}{4} = \frac{5\pi}{2}$
 $-\frac{\pi}{4}, \frac{9\pi}{4}, \frac{19\pi}{4}, \frac{29\pi}{4}, \frac{39\pi}{4}, \frac{49\pi}{4}, \frac{59\pi}{4}, \frac{69\pi}{4}, \frac{79\pi}{4}$



Unit 3 (4.6) Horizontal Transformations of Trig. Functions

16. Write the equations for the given graphs



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Extra Practice:

Describe the transformation of each function.

1. $y = \sin 2\theta$ HD = $\frac{1}{2}$

H.D. $\frac{1}{2}$

$P = \frac{360}{2} = 180^\circ$

2. $y = \cos \frac{1}{2} \left(x - \frac{\pi}{3} \right)$

H.D. 2

H.T. $\frac{\pi}{3}$

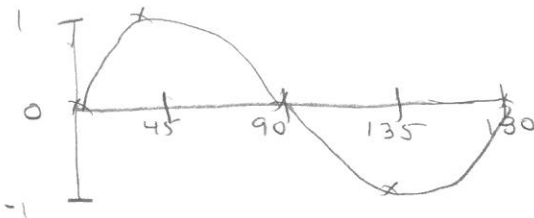
3. $y = \sin 3 \left(x + \frac{\pi}{4} \right)$

H.D. = $\frac{1}{3}$ HT $-\frac{\pi}{4}$

$P = \frac{2\pi}{3}$

4. Graph one cycle of question #1. $\frac{180}{4} = 45^\circ$

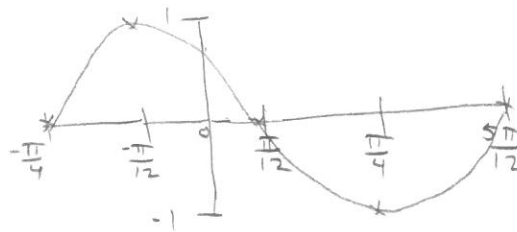
0, 45, 90, 135, 180



5. Graph one cycle of question #3.

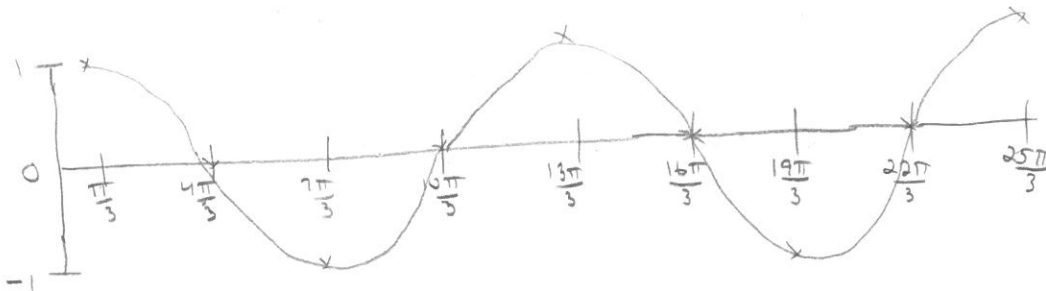
Start at $-\frac{\pi}{4}$ or $-\frac{3\pi}{12}$

$\frac{2\pi}{3} = 4 = \frac{2\pi}{12} = \frac{\pi}{6}$ or $\frac{2\pi}{12}$
 $-\frac{3\pi}{12}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}$



6. Graph two cycles of question #2. $P = \frac{2\pi}{\frac{1}{2}} = 4\pi$ $\frac{4\pi}{4} = \pi$ or $\frac{3\pi}{3}$

$\frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}, \frac{13\pi}{3}, \frac{16\pi}{3}, \frac{19\pi}{3}, \frac{22\pi}{3}, \frac{25\pi}{3}$



Write an equation for each of the following set of transformations.

7. The sine function with a horizontal dilation of 4 and a horizontal translation of -60° .

$$y = \sin \left[\frac{1}{4} (\theta + 60^\circ) \right]$$

8. The cosine function with a horizontal dilation of $\frac{1}{3}$ and a horizontal translation of π .

$$y = \cos 3(x - \pi)$$