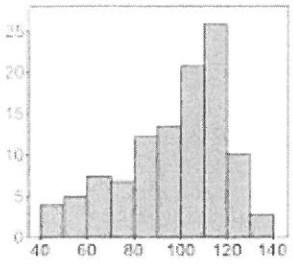
Unit 4 1.5

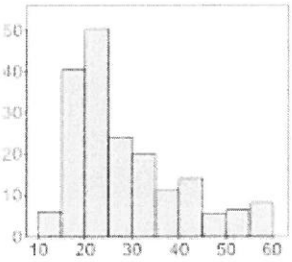
Normal Distributions

Data can be "distributed" (spread out) in many different ways. See examples below:

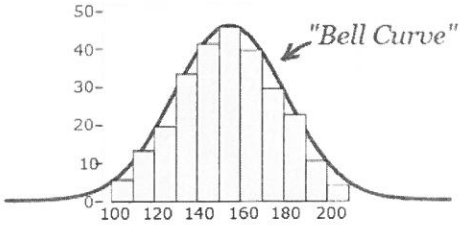
It can be spread out more on the left



Or more on the right



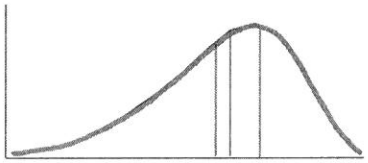
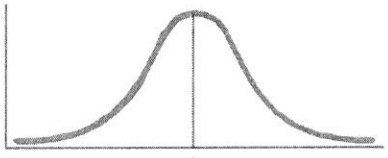
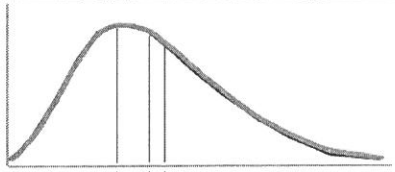
Or symmetric



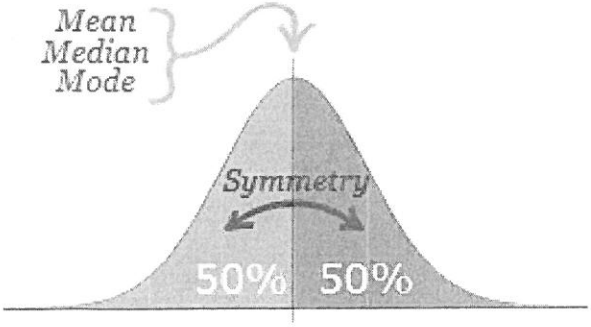
- There are many cases where the data tends to be around a central value with no bias left or right. The "Bell Curve" is a Normal Distribution. It is called this because it looks like a bell.
  - Examples: Heights of people, blood pressure, and IQ

Skewness

- A distribution of data is skewed if it is not symmetric and extends more to one side than the other.

Skewed to the left (negatively skewed)	Symmetric (zero skewness)	Skewed to the right (positively skewed)
 **SKEWED LEFT** (negatively)	 **SYMMETRIC**	 **SKEWED RIGHT** (positively)

Our focus will be on "normal distribution".



The **Normal Distribution** has:

- mean = median = mode
- symmetry about the center
- 50% of values less than the mean and 50% greater than the mean

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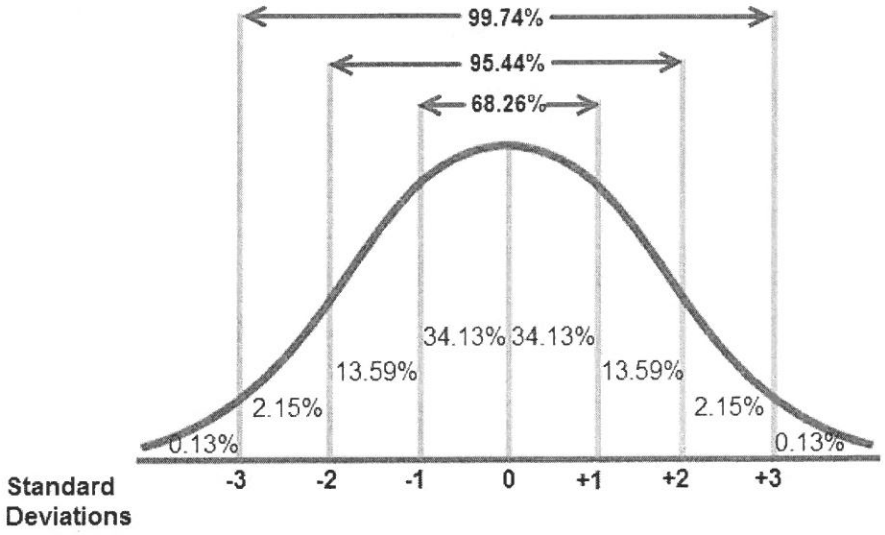
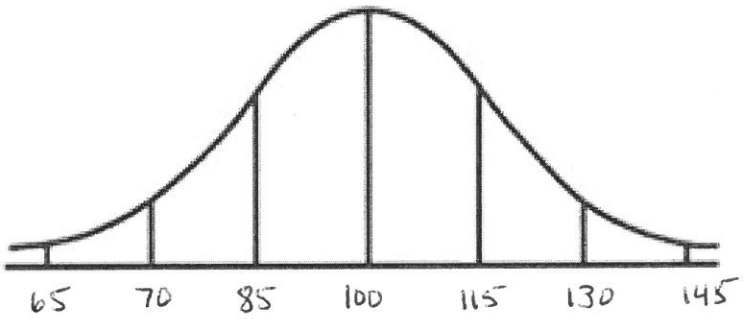
Standard Deviation is a key component in Normal Distribution.



Examples:

The "average" (mean) IQ is 100 and it has a standard deviation of 15. LABEL THE CURVE



A set of data with a mean of 45 and a standard deviation of 8 is normally distributed. Find each value, given its distance from the mean.

1. +1 standard deviation from the mean

$45+8=53$

2. −2 standard deviations from the mean

$45-2(8)=29$

A set of data with a mean of 77 and a standard deviation of 6.5 is normally distributed. Find each value, given its distance from the mean.

1. +3 standard deviations from the mean

$77+3(6.5)=96.5$

2. −1 standard deviation from the mean

$77-1(6.5)=70.5$

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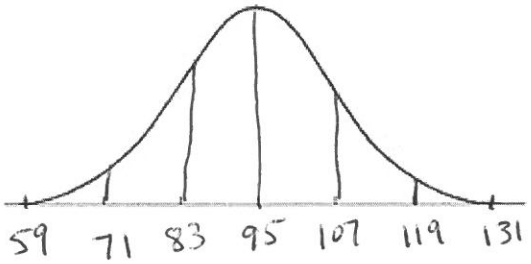
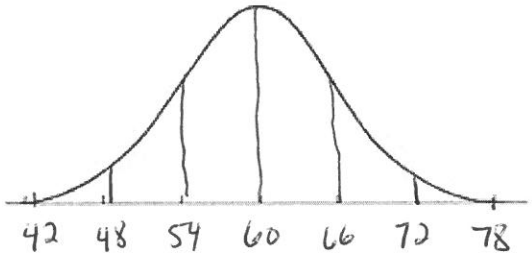
Sketch a normal curve for each distribution. Label the x-axis at ± 1, ± 2, and ± 3 standard deviations from the mean.

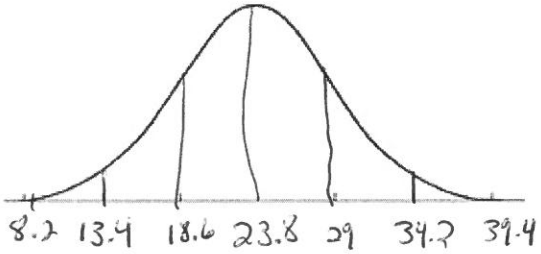
1. $mean = 95$; standard deviation $= 12$



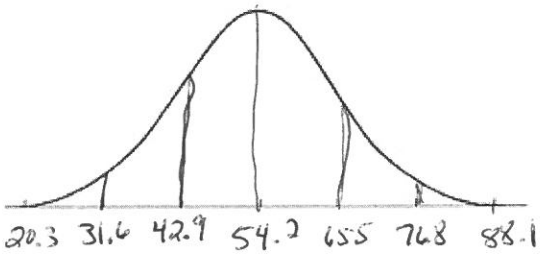
2. $mean = 60$; standard deviation $= 6$



3. $mean = 23.8$; standard deviation $= 5.2$



4. $mean = 54.2$; standard deviation $= 11.3$



5. A set of data has a normal distribution with a mean of 5.1 and a standard deviation of 0.9.
   a. Sketch a normal curve for the distribution.
   b. Find the percent of data within the interal of 6.0 and 6.9.

      13.59%

   c. Find the percent of data greater than 6.9.

      2.15 + 0.13 = 2.28%

   d. Find the percent of data within the interal of 4.2 and 6.0.

      68.26%

   e. Find the percent of data less than 4.2.

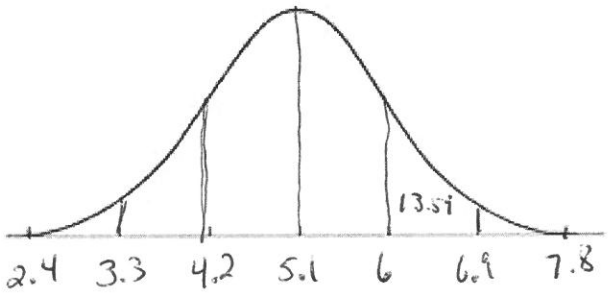
      .5 − .3413 = 15.87%

   f. Find the percent of data less than 5.1.

      50%

   g. Find the percent of data within the interal of 4.2 and 5.1.

      34.13%

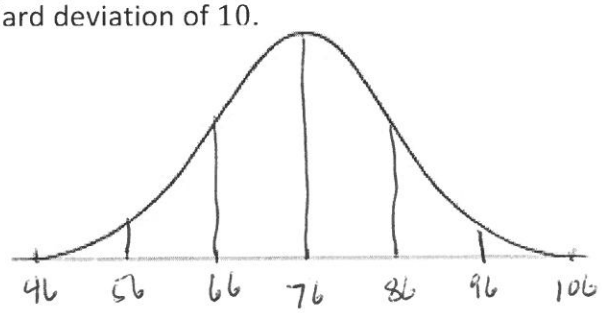


6. Test scores are normally distributed with a mean of 76 and a standard deviation of 10.
   a. In a group of 230 tests, how many students score above 96?

      2.15 + 0.13 = 2.28% 230 * .228 = 52

   b. In a group of 230 tests, how many students score below 66?

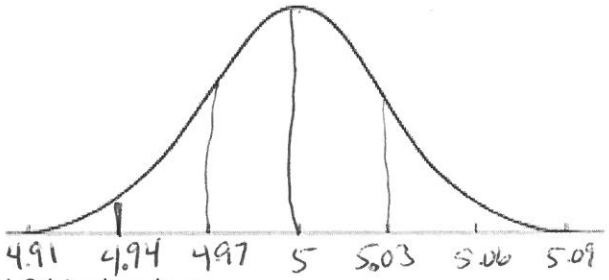
      .5 − .3413 = 15.87% 230 * .1587 = 37



   c. In a group of 230 tests, how many students score within one standard deviation of the mean?

      230 * .6826 = 157

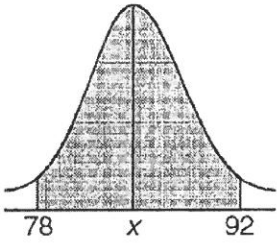
7. The number of nails of a given length is normally distributed with a mean length of 5.00 inches and a standard deviation of 0.03 inches.



   a. Find the number of nails in a bag of 120 that are less than 4.94 inches long.
   120 * .228 = 27
   b. Find the number of nails in a bag of 120 that are between 4.97 and 5.03 inches long.
   120 * .6826 = 82
   c. Find the number of nails in a bag of 120 that are over 5.03 inches long.
   .5 - .3413 = .1587 120 * .1587 = 19

8. In the diagram, the shaded area represents approximately 95% of the scores on a standardized test. If these scores ranged from 78 to 92,

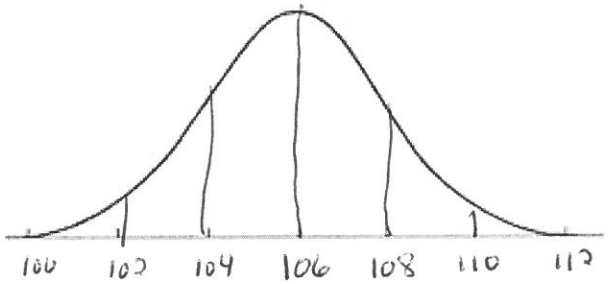
   a. What is the mean? 85
   b. What is the standard deviation? 3.5



x + 2sd = 92
x - 2sd = 78
$\frac{78+92}{2} = 85$
$\frac{92-85}{2} =$

9. A machine is used to put bolts into boxes. It does so such that the actual number of bolts in a box is normally distributed with a mean of 106 and a standard deviation of 2.

   a. Draw and label the normal curve from the information.



   b. What percentage of boxes contain more than 104 bolts?
   .5 + .3413 = 84.13%
   c. What percentage of boxes contain more than 110 bolts?
   .0215 + .0013 = .0228 2.28%
   d. What percentage of boxes contain less than 108 bolts?
   .5 + .3413 = 84.13%
   e. What percentage of boxes contain less than 108 bolts?
   13.59 + 34.13 + 34.13 + 13.59 + 2.15
   f. What percentage of boxes contains between 102 and 112 bolts?
   97.59%
   g. What percentage of boxes contains between 100 and 106 bolts?
   99.74/2 = 49.87%

10. On a standardized test, Phyllis scored 84, exactly one standard deviation above the mean. If the standard deviation for the test is 6, what is the mean score for the test? 84 - 6 = 78

11. The heights of a group of girls are normally distributed with a mean of 66 inches. If 95% of the heights of these girls are beween 63 and 69 inches, what is the standard deviation for this group?
   $\frac{69-66}{2}$ = 1.5 in

12. Battery lifetime is normally distributed for large samples. The mean lifetime is 500 days and the standard deviation is 61 days. About what percent of batteries have lifetimes longer than 561 days?
   .5 - .3413 = 15.87 about 16%