Unit 2 (1.7) **Natural Logarithms and the Base *e* - Student Notes**

Warm-Up: Use any method to solve

1. $64 = 8^{x-6}$
2. $7^x = 54$
3. $9^{-x+3} = 27^{x-3}$
4. $16^{2x} = 9^{3x-5}$

**EXAMPLE**

Suppose you deposit $1000 in an account paying 5% annual interest compounded continuously. Using the formula $y = Pe^{rt}$, find:

a. the balance after 10 years.

b. How long it will take for the balance in your account to reach at least $1500?

## Base *e*

The base *e* is an irrational number much like π. It's numerical value is 2.71828………., but it's much easier to use in its variable form (see button for $e^x$ on your calculator). **Natural base exponential functions**, such as the equation in the above example, are used extensively in science to model quantities that grow and decay continuously, and in banking for continuously compounded interest.

Practice:

1. $e^2$
2. $e^{-1.3}$

# The Natural Logarithm

**The logarithm with base $e$ is called the natural logarithm ($log_e x$), but is most often abbreviated $ln\, x$.**

- see button on calculator and try: $ln4$ and $ln0.05$

IMPORTANT: $e^x$ and the $ln\, x$ have the same properties as other exponents and logarithms

<table>
<tr>
<td>Property of Equality<br><br>$\boldsymbol{lnx = lny}$<br><br>Example: $\ln(x-7) = ln3$<br>$x - 7 = 3$<br>$x = 10$</td>
<td>Product Property<br><br>$\boldsymbol{ln(cd) = ln(c) + ln(d)}$<br><br>Example: $ln4 + lnx = ln(4x)$</td>
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<td>Quotient Property<br><br>$\boldsymbol{ln\left(\frac{c}{d}\right) = ln(c) - ln(d)}$<br><br>Example: $lnx - ln3 = ln(\frac{x}{3})$</td>
<td>Power Property<br><br>$\boldsymbol{ln(m)^p = p * ln(m)}$<br><br>Examples: $ln2^x = 7$<br>$x * ln2 = 7$<br>$x = \frac{7}{ln2}$</td>
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1. Evaluate each expression or solve.

a. $e^{ln7}$

b. $lne^{4x+3}$

c. $\ln(x-7) = 2$

d. $ln7 + lnx = ln28$

e. $\ln(x+8) - \ln(7) = 3$

f. $2lnx + ln4 = ln100$

g. $4e^{x+5} + 7 = 35$

h. $2e^{0.5x} = 26$

2. (the opening example) Suppose you deposit \$1000 in an account paying 5% annual interest compounded continuously. Using the formula $y = Pe^{rt}$, find:
   a. the balance after 25 years.

   b. How long it will take for the balance in your account to reach at least \$1500?

3. If you deposit \$2025 in a savings account paying 3.2% interest compounded continuously, how much money will you have after 15 years? How long would it take you to triple your money?