Unit 2 (1.7) **Natural Logarithms and the Base *e* - Student Notes**

Warm-Up: Use any method to solve

1. $64 = 8^{x-6}$

$8^2 = 8^{x-6}$

$2 = x - 6$

$8 = x$

2. $7^x = 54$

$\log 7^x = \log 54$

$x \log 7 = \log 54$

$x = 2.05$

3. $9^{-x+3} = 27^{x-3}$

$3^{2(-x+3)} = 3^{3(x-3)}$

$-2x + 6 = 3x - 9$

$15 = 5x$

$x = 3$

4. $16^{2x} = 9^{3x-5}$

$2x \log 16 = (3x-5) \log 9$

$2.52x = 3x - 5$

$-.47x = -5$

$x = 10.50$

EXAMPLE

Suppose you deposit \$1000 in an account paying 5% annual interest compounded continuously. Using the formula $A = Pe^{rt}$, find:

a. the balance after 10 years. $A = 1000e^{.05(10)}$ \$1648.72

b. How long it will take for the balance in your account to reach at least \$1500?

$1500 = 1000e^{.05t}$

$1.5 = e^{.05t}$?

Base *e*

The base *e* is an irrational number much like π. It's numerical value is 2.71828..........., but it's much easier to use in its variable form (see button for e^x on your calculator). **Natural base exponential functions**, such as the equation in the above example, are used extensively in science to model quantities that grow and decay continuously, and in banking for continuously compounded interest.

Practice:

1. e^2 7.39

2. $e^{-1.3}$ .27

The Natural Logarithm

The logarithm with base *e* is called the natural logarithm ($log_e x$), but is most often abbreviated $ln\, x$.

- see button on calculator and try: $ln4$ and $ln0.05$

IMPORTANT: e^x and the $ln\, x$ have the same properties as other exponents and logarithms

Property of Equality	Product Property
$\boldsymbol{lnx = lny}$ Example: $\ln(x-7) = ln3$ $x - 7 = 3$ $x = 10$	$\boldsymbol{ln(cd) = ln(c) + ln(d)}$ Example: $ln4 + lnx = ln(4x)$
Quotient Property	**Power Property**
$\boldsymbol{ln\left(\frac{c}{d}\right) = ln(c) - ln(d)}$ Example: $lnx - ln3 = ln(\frac{x}{3})$	$\boldsymbol{ln(m)^p = p * ln(m)}$ Examples: $ln2^x = 7$ $x * ln2 = 7$ $x = \frac{7}{ln2}$

1. Evaluate each expression or solve.

a. e^{ln7}

7

b. lne^{4x+3}

$4x+3$

c. $\ln(x-7) = 2$

$e^2 = x - 7$

$e^2 + 7 = x$

$x = 14.39$

d. $ln7 + lnx = ln28$

$\ln 7x = \ln 28$

$7x = 28$

$x = 4$

e. $\ln(x+8) - \ln(7) = 3$

$\ln \frac{x+8}{7} = 3$

$e^3 = \frac{x+8}{7}$

$7e^3 = x + 8$

-8

$x = 132.60$

f. $2lnx + ln4 = ln100$

$\ln x^2 + \ln 4 = \ln 100$

$\ln 4x^2 = \ln 100$

$4x^2 = 100$

$x^2 = 25$

$x = \pm 5$ $x = 5$

g. $4e^{x+5} + 7 = 35$

$4e^{x+5} = 28$

$e^{x+5} = 7$

$x+5 = \ln 7$

-5

$x = -3.05$

h. $2e^{0.5x} = 26$

$e^{0.5x} = 13$

$0.5x = \ln 13$

$\div 0.5$

$x = \cancel{}$ 5.13

2. (the opening example) Suppose you deposit \$1000 in an account paying 5% annual interest compounded continuously. Using the formula $y = Pe^{rt}$, find:

a. the balance after 25 years.

$1000e^{.05(25)}$

$1000e^{1.25}$

\$3490.34

b. How long it will take for the balance in your account to reach at least \$1500?

$1500 = 1000e^{.05t}$

$1.5 = e^{.05t}$

$\ln 1.5 = .05t$

$\div .05$

$t \approx 8.1$ hr

3. If you deposit \$2025 in a savings account paying 3.2% interest compounded continuously, how much money will you have after 15 years? How long would it take you to triple your money?

$y = 2025e^{.032(15)}$

$2025e^{.48}$

\$3272.55

$6075 = 2025e^{.032t}$

$3 = e^{.032t}$

$\ln 3 = .032t$

$\div .032$

$t \approx 34.33$ yrs