Unit 4 1.7

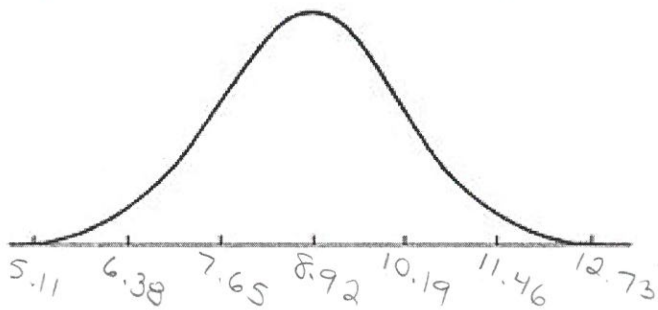
Answers 1.7

Key

Normal Distributions: Finding Probabilities

Warm-Up:

1. RAVE cinemas studied its customers to determine how much money they spend on concessions. The study revealed that the spending is approximately normally distributed with a mean of $8.92 and a standard deviation of $1.27. Label the normal curve below using the given information.



a. What percentage of customers spend between $7.65 and $11.46? 81.85%

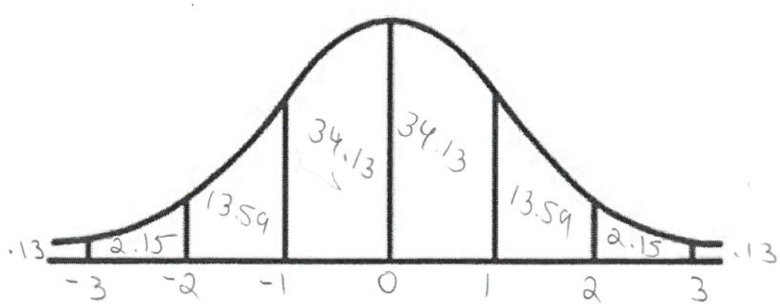
b. What is the probability that a customer spends at least $10.19? .1587

Notes:

- Percentages are equivalent to finding proportions or probabilities.
- The same process in finding a percentage can be used to find probability, proportion, or percent.
  - An event that has a probability of 0.40 is the same as a proportion of 40/100, or 40%.

When we are finding the percent, probability, or proportion, we are finding the area of a specific region under the curve.

- ❖ Label the normal curve below with the appropriate percentages.
- ❖ Convert the percentages to decimals then add up all of your decimals.
  - What do they add up to? 1 ← 100%
  - The area under the entire curve is 1



2. See Warm-Up example: What is the probability that a customer spends at least $9.00? Can't figure yet

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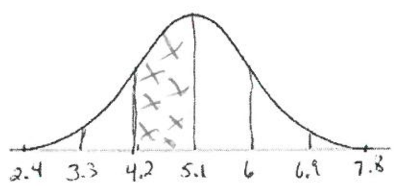
- In real world problems, the number of standard deviations away from the mean is rarely a whole number.
  - The number of standard deviations away from the mean is often referred as the standard score or <u>"z-score"</u>.
- You are going to learn how to find these standard scores and learn how to read the table needed to find the corresponding percentages (proportions, probabilities).

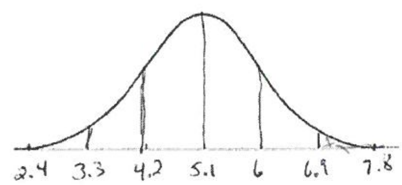
❖ In an earlier problem, the set of data was normally distributed with a mean of 5.1 and a standard deviation of 0.9. Find the percent of data within each interval given.

1. Between 4.2 and 5.1



34.13%

2. Greater than 6.9



2.28%

Standard Score (z-score) $z = \frac{x-\mu}{\sigma}$

"z" is the number of standard deviations above or below the mean
"x" is the data point
μ is the mean
σ is the standard deviation

- A negative z-score is below the mean
- A positive z-score is above the mean

For example:

- What is the z-score of 4.2?
  - $z = \frac{4.2-5.1}{0.9}$ would result in a z-score of -1 or one standard deviation below the mean.

- What is the z-score of ~~5.1~~ 6.9?
  - $z = \frac{6.9-5.1}{0.9}$ would result in a z-score of 2 or two standard deviations above the mean.

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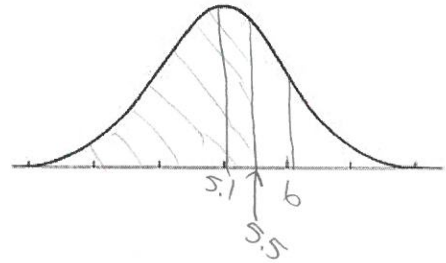
- What if we wanted to know the percentage of scores that are below 5.5?
  - Draw the normal curve to determine what you are trying to find.
  - Find the z-score using the formula above. from ~~last page~~ Previous page



$$Z = \frac{5.5-5.1}{0.9} = .44$$

  - Round to the nearest hundredths place.
  - Since this number does not fit into the 68-95-99 normal curve for 1, 2, or 3 standard deviations from the mean we need to use the table "A-2" to find the percentage.

Using Table A-2 (Area to the LEFT of the data point)

1. Using the z-score calculated above break the number into tenths and hundredths.
   a. Using the tenths place look in the left hand column until you reach the desired number.
   b. Using the hundredths place look across the top until you reach the desired number.
   c. Where these two numbers intersect in the table is your answer.

➢ You should have found .6700 which means that 67% of the scores are below 5.5.

Directions:

1. Draw the normal curve.
   a. Drawing the curve is very important. Sometimes the question will ask for the area to the left of a score, sometimes it will ask for the area to the right of a score, and sometimes it will ask for the area between two scores. Always draw the normal curve and shade the appropriate area.
      i. Table A-2 always gives the area to the left of the score.
      ii. To find the area to the right of the score take the number found in A-2 and subtract it from one.
      iii. To find the area between two scores subtract ~~the number found in A-2 from 0.5 or subtract the percentages away from each other~~.
2. Use the formula $z = \frac{x-\mu}{\sigma}$ to find the z-score, round to the hundredths place.
3. Break the z-score into the tenths place and hundredths place.
4. Find the tenths place along the left hand side of the table and the hundredths place along the top.
5. Where these two numbers intersect is your answer.

Area = percent = probability = proportion

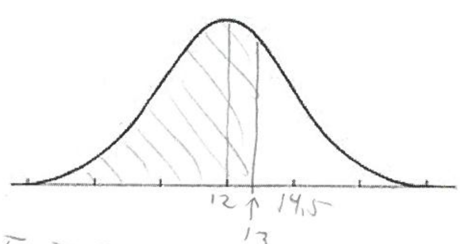
Examples:

Mean = 12, standard deviation = 2.5.

What are the percent of scores: $Z = \frac{13-12}{2.5}$

a. below 13? $Z = .40$

.6554



65.54%

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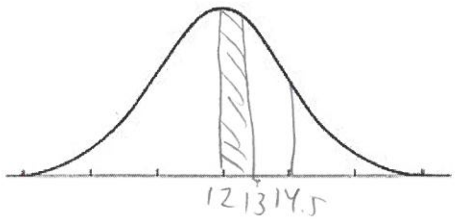
b. between 12 and 13?

Subtract the 2 %s

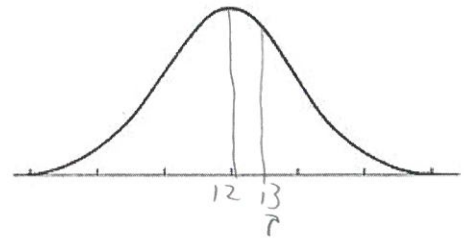


$$z = \frac{13-12}{2.5}$$

.6554
− .5
.1554

15.54%

c. above 13?



Below 13 was .6554

1 − .6554 = .3446

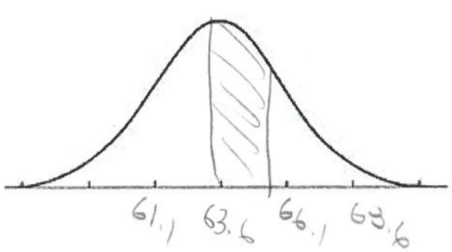
34.46%

Standard Score (z-score) $z = \frac{x-\mu}{\sigma}$

"z" is the number of standard deviations above or below the mean
"x" is the data point
μ is the mean
σ is the standard deviation

In exercises 1-5 assume that women's heights are normally distributed with a mean given by $\mu = 63.6\ in$ and a standard deviation given by $\sigma = 2.5\ in$. Also assume that a woman is randomly selected. Draw a graph and find the indicated probability.

1. P(63.6 in. < x < 65.0 in.)

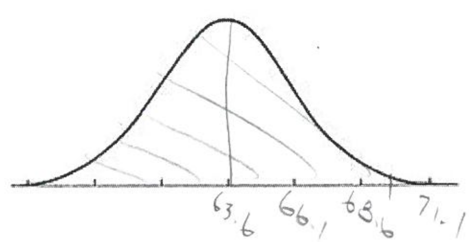


$$\frac{65-63.6}{2.5} = .56$$

.7123
− .5
.2123

21.23%

2. P(x < 70.0 in.)

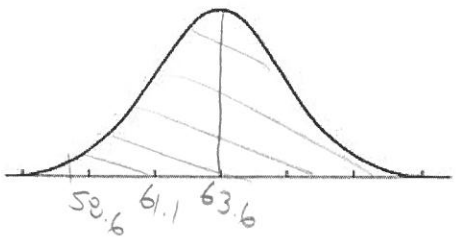


$$\frac{70-63.6}{2.5} = 2.56$$

.9948

99.48%

3. P(x > 58.1 in.)

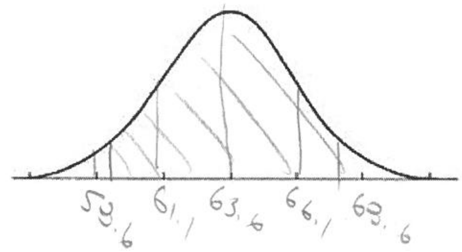


$$\frac{58.1-63.6}{2.5} = -2.2$$

.0139 $\quad$ 1 − .0139 = .9861

98.61%

4. P(59.1 in. < x < 66.6 in.)

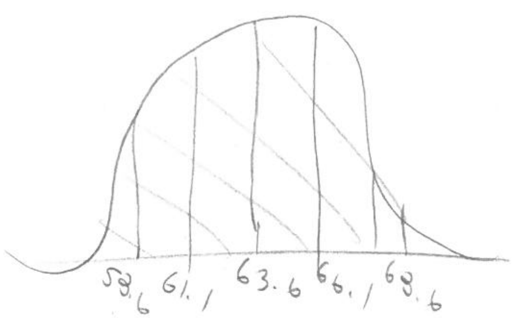


$$\frac{59.1-63.6}{2.5} = -1.8 \qquad \frac{66.6-63.6}{2.5} = 1.2$$

.0359 $\quad$ .8849

.8849 − .0359 = .849 $\quad$ 84.9%

5. The Beanstalk Club, a social organization for tall people, has a requirement that women must be at least 70 in. tall. Suppose you are considering opening a branch of the Beanstalk Club in a metropolitan area with 500,000 adult women. ($\mu = 63.6\ in$ and $\sigma = 2.5\ in.$)



a. Find the percentage of adult women who are eligible for membership because they meet the minimum height requirement.

$$\frac{70-63.6}{2.5} = 2.56$$

.9948 $\quad$ 1 − .9948 $\quad$ .0052 $\quad$ .52%

b. Among the 500,000 adult women living in this metropolitan area, how many are eligible for Bean Stalk membership?

.0052 × 500000 $\quad$ 2600 members

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6. Replacement times for TV sets are normally distributed with a mean of 8.2 years and a standard deviation of 1.1 years. Find the probability that a randomly selected TV set will have a replacement time less than 7.0 years.

$\frac{7-8.2}{1.1}$ -1.09

.1379

13.79%

7. Replacement times for computer printers are normally distributed with a mean of 7.1 years and a standard deviation of 1.4 years. Find the probability that a randomly selected computer printer will have a replacement time more than 8.0 years.

$\frac{8-7.1}{1.4} = .64$

.7389

$1-.7389 = .2611$

26.11%

8. Assume the weights of paper discarded by households each week are normally distributed with a mean of 9.4 lbs. and a standard deviation of 4.2 lbs. Find the probability of randomly selecting a household and getting one that discards between 5.0 lbs. and 8.0 lbs.

$\frac{5-9.4}{4.2} = -1.05$ $\frac{8-9.4}{4.2} = -.33$

.1469 .3707

$.3707-.1469 = .2238$

22.38%

9. The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days. If we stipulate that a baby is *premature* if born at least 3 weeks early, what percentage of babies are born prematurely?

21

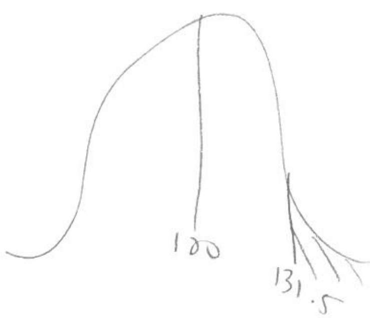
$268-21=247$

$$\frac{247-268}{15}=-1.4$$

.0808

8.08%

10. IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. Mensa is an organization for people with high IQs, and eligibility requires an IQ above 131.5.



$$\frac{131.5-100}{15}=2.1$$

.9821

$1-.9821=.0179$

1.79%

a. If someone is randomly selected, find the probability that he or she meets the Mensa requirement.

1.79%

b. In a typical region of 75,000 people, how many are eligible for Mensa?

$.0179 \times 75000$

1342.5

1342 people

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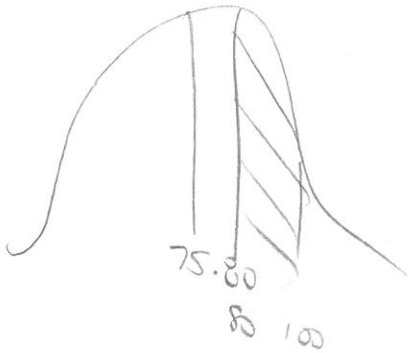
11. Girls age 13-17 spend an average of $75.80 per month on clothes. If the amounts are normally distributed with a standard deviation of $18.50. If a girl is randomly selected in that age category, what is the probability that she spends between $80 and $100 in one month?



$\frac{80-75.80}{18.50} = .23$ $\quad$ $\frac{100-75.80}{18.50} = 1.31$

.5910 $\quad$ .9049

$.9049 - .5910 = .3139$

31.39%

12. Some vending machines are designed so that their owners can adjust the weights of quarters that are accepted. If many counterfeit quarters are found, adjustments are made to reject more coins, with the effect that most of the counterfeit coins are rejected along with many legal coins. Assume that quarters have weights that are normally distributed with a mean of 5.67 g and a standard deviation of 0.070 g. If a vending machine is adjusted to reject quarters weighing less than 5.50 g or more than 5.80 g, what is the percentage of legal quarters that are rejected?

$\frac{5.5-5.67}{.07} = -2.43$ $\quad$ $\frac{5.8-5.67}{.07} = 1.86$

.0075 $\quad$ .9686

$1 - .9686$

.0314

$.0075 + .0314 =$

.0389

3.89% rejected