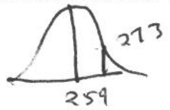
Normal Distributions: Finding Scores

Warm-Up:

1. Pleasant Valley University (PVU) accepts students who score at least a 273 on their entrance exam. The mean of the exam is 259 with a standard deviation of 18.4. What percentile must you be in to be accepted?



$z = \frac{273-259}{18.4} = .76$ TABLE A2 77.64%

2. To be accepted into the honors program you must score in the top 2%, what score do you have to get on the test to earn a spot in the program?

Look up .9800 in body of table closest is .9798

$\frac{x-259}{18.4} = 2.05$ $x = 297$

You will still be utilizing the same z-score formula. Only now you will find the z-score in the table and then need to use the formula to solve for x.

Standard Score (z-score) $z = \frac{x-\mu}{\sigma}$

"z" is the number of standard deviations above or below the mean

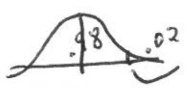
"x" is the data point

μ (mu) is the mean of the population

σ (sigma) is the standard deviation of the population

Finding a score (x): Use example #2 from the warm-up.

1. Draw a bell curve and determine the given probability then identify the *x* value(s) being sought.



2. Rewrite the probability (percentage) as a decimal. .9800
3. Find this decimal (or as close as possible) inside Table A-2
   - Remember that the table shows you the area to the LEFT. In this example we want the TOP 2%, so we are looking for 98% or .98 in our table
4. By working backward, determine the z-score for this probability. (tenths place down the left hand side and the hundredths place across the top)
   a. If the percentage is less than 50% you will have a negative z-score use the correct side of the table.
5. Using the z-score formula, $z = \frac{x-\mu}{\sigma}$, enter the values for $\mu, \sigma, and\ z$, then solve for x.

6. Refer to the sketch of the curve to verify that the solution makes sense in the context of the graph and the context of the problem.
   - You could also test your answer by plugging it into the formula for x and ensure that you get the correct z-score.

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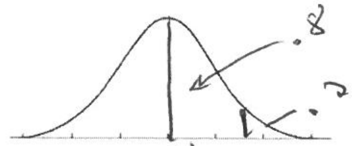
**** Previously you were asked to find the percent of data within a given interval. Now we are going to be given the percent of data in the given interval and find the appropriate score.**

Example: Let's say we want to find the **score** (x) that separates the bottom 80% from the top 20%. (P_{80}) For this particular set of data the scores are normally distributed with a mean of 5.1 and a standard deviation of 0.9.

1. Draw bell curve and label.

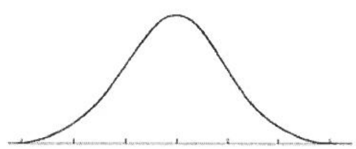


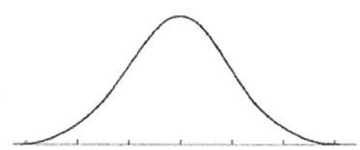
2. Use Table A-2 to find the area closest to __.8__ in the body of the table. The closest is __.7995__ which has a corresponding z-score of __.84__.
3. Now use the z-score formula $z = \frac{x-\mu}{\sigma}$ >> $.84 = \frac{x-5.1}{0.9}$ to get x=__5.86__.
   - Work backwards to get x.
4. P_{80}, the score that separates the bottom 80% from the top 20% is __5.86__.

When considering problems where you are finding scores given the probabilities, there are three important cautions to keep in mind:

1. *Don't confuse z-scores and areas* (probabilities, percentages).
   - Remember, *z* scores are distances along the horizontal scale, but areas represent regions under the normal curve.
   - Table A-2 lists *z* scores in the left column and across the top row but areas are found in the body of the table.

2. *Choose the correct (right/left) side of the graph.* A score separating the top 10% from the others is located on the right side of the curve, but a score separating the bottom 10% will be located on the left side of the curve.





3. *A z score must be negative whenever it is located to the left of the centerline (mean).*

In exercises 1-4 assume that women's heights are normally distributed with a mean given by $\mu = 63.6\ in$ and a standard deviation given by $\sigma = 2.5\ in$. Also assume that a woman is randomly selected. Find the height for the given percentile.

1. P_{85}

area .8508 → z score 1.04

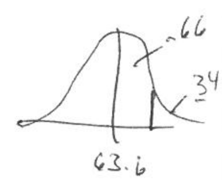
$$1.04 = \frac{x - 63.6}{2.5}$$

$$x = 66.2 \text{ in}$$

2. P_{66}

area .6591 → z score .41

$$.41 = \frac{x - 63.6}{2.5}$$



$$x = 64.63 \text{ in}$$

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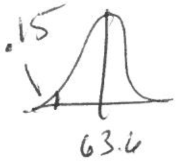
area $.1492 \rightarrow$ z score $= -1.04$

3. P_{15}

$$-1.04 = \frac{x - 63.6}{2.5}$$

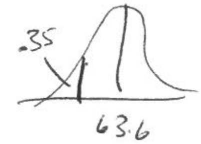


$x = 61$ in

area $.3483 \rightarrow$ z score $-.39$

4. P_{35}

$$-.39 = \frac{x - 63.6}{2.5}$$

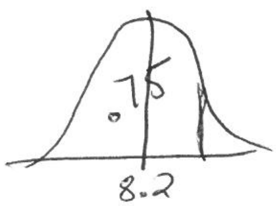


$x = 62.63$ in

5. Replacement times for TV sets are normally distributed with a mean of 8.2 years and a standard deviation of 1.1 years. Find the replacement time that separates the top 25% from the bottom 75%. This result would be helpful to an appliance company that wants to offer service contracts for TV sets.

P_{75}

$.7486$



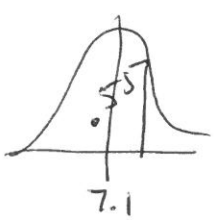
$$.67 = \frac{x - 8.2}{1.1}$$

$x = 8.94$ years

6. Replacement times for computer printers are normally distributed with a mean of 7.1 years and a standard deviation of 1.4 years. Find the replacement time that separates the top 45% from the bottom 55%.

P_{55}

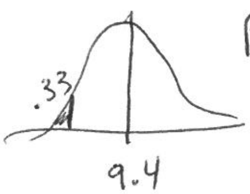
$.5517$



$$.13 = \frac{x - 7.1}{1.4}$$

$x = 7.28$ years

7. Assume the weights of paper discarded by households each week are normally distributed with a mean of 9.4 lbs. and a standard deviation of 4.2 lbs. Find the weight that separates the bottom 33% from the top 67%.



P_{33}

$.3300$

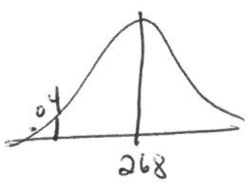
$$-.44 = \frac{x - 9.4}{4.2}$$

$x = 7.55$ lbs

8. The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days. If we stipulate that a baby is *premature* if the length of pregnancy is in the lowest 4%, find the length that separates the premature babies from those that are not. Premature babies often require special care, and this result could be helpful to hospital administrators in planning for that care.

P_4

$.0401$



$$-1.75 = \frac{x - 268}{15}$$

$x = 241.8$ days

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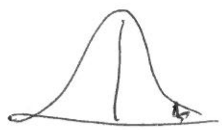
9. IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. If we define a genius to be someone in the top 1% of IQ scores, find the score separating the geniuses from the rest of us. This score could be used by a "think tank" company as one criterion for employment.

P_{99}

.9901



$$2.33 = \frac{x-100}{15}$$

$$x = 135$$

10. Girls age 13-17 spend an average of \$75.80 per month on clothes. The amounts are normally distributed with a standard deviation of \$18.50. What is the amount spent by girls in the 25 percentile?

P_{25}

.2514

$$-.67 = \frac{x-75.80}{18.50}$$

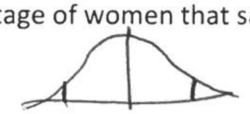
$$x = \$63.41$$

11. To be eligible for the U.S. Marine Corps, a woman must have a height between 59 in. and 73 in. Recall that heights of women are normally distributed with a mean of 63.6 in. and a standard deviation of 2.5 in.

   a. Find the percentage of women that satisfy that requirement.

$$\frac{59-63.6}{2.5} \qquad \frac{73-63.6}{2.5}$$

96.7%



$$.9999 - .0329 = .967$$

$$z = -1.84 \qquad z = 3.76$$

   b. If the requirement is changed to exclude the shortest 1% and the tallest 1%, find the heights that are acceptable.

$57.78 < x < 69.43$

P_1 .0099 −2.33

P_{99} .9901 2.33

$$-2.33 = \frac{x-63.6}{2.5} \qquad 2.33 = \frac{x-63.6}{2.5}$$

$$x = 57.78 \text{ in} \qquad x = 69.43 \text{ in}$$

12. A teacher gives a test and gets normally distributed results with a mean of 50 and a standard deviation of 10. If grades are assigned according to the following scale, find the numerical limits for each letter grade.

A: top 10% P_{90} .8997

$$1.28 = \frac{x-50}{10} \qquad x = 62.8$$

B: scores above the bottom 70% and below the top 10%

P_{70} .6985

$$.52 = \frac{x-50}{10} \qquad 55.2 < x < 62.8$$

C: scores above the bottom 30% and below the top 30%

P_{30} .3015

$$-.52 = \frac{x-50}{10} \qquad 44.8 < x < 55.2$$

D: scores above the bottom 10% and below the top 70%

P_{10} .1003

$$-1.28 = \frac{x-50}{10} \qquad 37.2 < x < 44.8$$

F: bottom 1% 10%

$$x < 37.2$$