

Solving Radical Equations – Student Notes

Solve the following. Check your solutions.

1. $\sqrt{x} = 5$
 $x = 25$

2. $\sqrt{x} = -5$
 \emptyset

3. $\sqrt{x} - 6 = 4$
 $\sqrt{x} = 10$
 $x = 100$

4. $\sqrt[3]{x} - 6 = 4$
 $\sqrt[3]{x} = 10$
 $x = 1000$

5. $\sqrt{x} + 6 = 4$
 $\sqrt{x} = -2$
 \emptyset

6. $\sqrt[3]{x} + 6 = 4$
 $\sqrt[3]{x} = -2$
 $x = -8$

What happened in #5?

No solution

Why do you suppose that happened?

$$\sqrt{x} \neq -\#$$

What do we call solutions that appear to work, but don't work in the original equation?

extraneous

What other types of equations sometimes have these types of solutions?

rational

Why did it work in #6?

$\sqrt[3]{x} \text{ can } = -\#$
 $(-2)(-2)(-2) = -8$ so $\sqrt[3]{-8} = -2$

Examples:

1. $\sqrt{2x-3} = 11$

$$2x-3=121$$

$$2x=124$$

$$x=62$$

2. $\sqrt[3]{6-x} = -3$

$$6-x = -27$$

$$-x = -33$$

$$x=33$$

3. $\sqrt{3x+5} - 2 = -1$

$$\sqrt{3x+5} = 1$$

$$3x+5=1$$

$$3x = -4$$

$$x = -\frac{4}{3}$$

Steps for solving radical equations:

- ① Isolate the radical
- ② $2, 3, \dots$ etc both sides to eliminate the $\sqrt{\quad}$
- ③ Solve for x
- ④ Check for extraneous solutions

* If $\sqrt{x} = -\#$, No solution

4. $\sqrt{x+5} - 9 = -12$

$$\sqrt{x+5} = -3$$

~~x~~

$$5. \sqrt{x^2 - 5x - 6} - 8 = 4$$

$$x^2 - 5x - 6 = 144$$

$$x^2 - 5x - 150 = 0$$

$$(x - 15)(x + 10) = 0$$

$$x = 15 \quad x = -10$$

$$6. -12\sqrt{x-6} = 18$$

$$\sqrt{x-6} = -1.5$$

\emptyset

$$7. 3\sqrt[3]{x+2} = 12$$

$$\sqrt[3]{x+2} = 4$$

$$x+2 = 64$$

$$x = 62$$

$$8. 3\sqrt{6-x} + 4 = 13$$

$$3\sqrt{6-x} = 9$$

$$\sqrt{6-x} = 3$$

$$6-x = 9$$

$$-x = 3$$

$$x = -3$$

$$9. \sqrt{6x-4} = \sqrt{2x+10}$$

$$6x-4 = 2x+10$$

$$4x = 14$$

$$x = \frac{7}{2}$$

$$10. \sqrt{9x-4} = \sqrt{7x-20}$$

$$9x-4 = 7x-20$$

$$2x = -16$$

$$x = -8$$

\emptyset

$$11. \sqrt{x-8} + \sqrt{x} = 2$$

$$\sqrt{x-8} = 2 - \sqrt{x}$$

$$x-8 = (2-\sqrt{x})(2-\sqrt{x})$$

$$x-8 = 4 - 2\sqrt{x} - 2\sqrt{x} + x$$

$$-12 = -4\sqrt{x}$$

$$3 = \sqrt{x}$$

$$9 = x \quad \emptyset$$

Check:

$$\sqrt{9-8} + \sqrt{9} = 2$$

$$1 + 3 \neq 2$$

$$12. \sqrt{x+1} = \sqrt{x+6} - 1$$

$$x+1 = (\sqrt{x+6}-1)(\sqrt{x+6}-1)$$

$$x+1 = x+6 - \sqrt{x+6} - \sqrt{x+6} + 1$$

$$x+1 = x+6 - 2\sqrt{x+6} + 1$$

$$-6 = -2\sqrt{x+6}$$

$$3 = \sqrt{x+6}$$

$$9 = x+6$$

$$3 = x$$