Alg2 Semester 1 Final Exam Review Name: _____ Period: _____

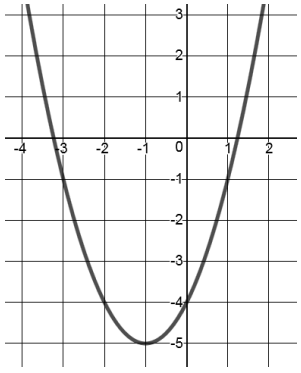
1. Explain how to find the vertex of a parabola if it is in vertex form. $y = a(x-h)^2 + k$

2. Explain how to find the x-intercepts of a parabola if it is in factored form. $y = a(x-p)(x-q)$

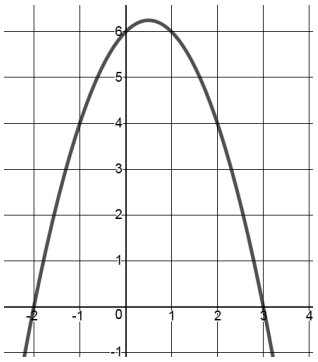
3. Explain how to find the y-intercept of a parabola if it is in standard form. $y = ax^2 + bx + c$

   a. Can you apply the same method to any equation to find the y-intercept?

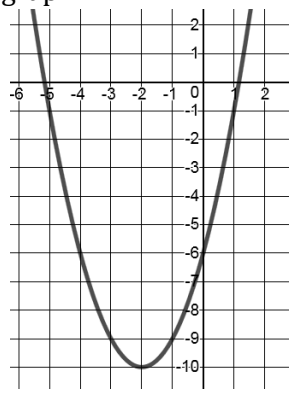
4. Match the equation to the graph.
   a. $y = (x+1)^2 - 5$
   b. $y = (x-1)^2 - 5$
   c. $y = -(x+1)^2 - 5$
   d. $y = -(x-1)^2 - 5$



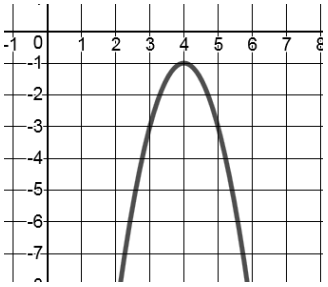
5. Match the equation to the graph.
   a. $y = (x-2)(x-3)$
   b. $y = (x+2)(x-3)$
   c. $y = -(x-2)(x+3)$
   d. $y = -(x+2)(x-3)$



6. Match the equation to the graph.
   a. $y = x^2 + 4x + 6$
   b. $y = -x^2 + 4x - 6$
   c. $y = x^2 + 4x - 6$
   d. $y = -x^2 + 4x + 6$



7. Match the equation to the graph.
   a. $y = -2(x+4)^2 - 1$
   b. $y = -2(x-4)^2 - 1$
   c. $y = 2(x-4)^2 + 1$
   d. $y = 2(x+4)^2 + 1$



8. Without using your graphing calculator determine the vertex.

$$y = -\frac{4}{5}(x-16)^2 + 27$$

9. Without using your graphing calculator determine the x-intercepts.

$$y = 6x^2 + x - 35$$

10. Without using your graphing calculator determine the y-intercept.

$$y = -81x^2 + 145x - 72$$

11. Without using your graphing calculator determine the x-intercepts.

$$y = (4x - 1)(2x + 9)$$

For questions 12-17, identify the key features of each equation. You may use a graphing calculator.

12. $y = 2x^2 - 3x + 4$

Vertex: _____

Y-intercept: _____

X-intercept(s): _____

13. $y = -x^2 - 7x + 8$

Vertex: _____

Y-intercept: _____

X-intercept(s): _____

14. $y = 5(x - 2)^2 - 1$

Vertex: _____

Y-intercept: _____

X-intercept(s): _____

15. $y = -2(x + 7)^2 + 3$

Vertex: _____

Y-intercept: _____

X-intercept(s): _____

16. $y = -(x + 2)(x - 6)$

Vertex: _____

Y-intercept: _____

X-intercept(s): _____

17. $y = 2(x - 4)(x + 4)$

Vertex: _____

Y-intercept: _____

X-intercept(s): _____

18. Determine the number and type of solutions. (Number of real and number of imaginary)

a. $y = -3x^2 + 15x - 8$

b. $y = 6x^2 - 12x + 11$

c. $y = \frac{1}{2}x^2 + 2x + 2$

19. To show his love for his wife on Valentine's Day, Mr. Sacco climbed to the top of an 84 foot cliff and launched a set of fireworks at a velocity of 160 feet per second.
(round all answers to the hundredths place)
The equation below represents the situation:

$$h = -16x^2 + 160x + 84$$

a. How high did the fireworks get in the sky?

b. How long did it take the fireworks to reach that height

c. How long were the fireworks in the air?

d. Use the equation to find out how high the fireworks were after 8 seconds.

e. When else would it be at that exact height?

20. Ms. Boehl has taken her math textbook to the roof of her house and looks down at the ravine which is 52 feet below. Ms. Boehl launches her book up into the air with an initial velocity of 96 feet per second. **(round all answers to the hundredths place)**

$$y = -16x^2 + 96x + 52$$

a. How long is the textbook in the air?

b. What is the maximum height the textbook will reach?

c. How long will it take for the textbook to reach the maximum height?

d. What is the height of textbook after 4 seconds?

e. At what time(s) will the textbook be 150 feet in the air?

Use the following expressions for questions 21-26 to perform the indicated operations.

$\boldsymbol{A}: (\boldsymbol{-4+3i})$ $\boldsymbol{B}: (\boldsymbol{8-2i})$ $\boldsymbol{C}: (\boldsymbol{-9-i})$

21. $A + C$

22. $B - A$

23. $C \cdot A$

24. $A - C$

25. $A \cdot B$

26. $B + A$

For questions 27-30, simplify.

27. i^{27}

28. i^{20}

29. i^{41}

30. i^{70}

Use the following expressions for questions 31-36 to perform the indicated operations.

$\boldsymbol{A}: (\boldsymbol{4x+1})$ $\boldsymbol{B}: (\boldsymbol{-3x-2})$ $\boldsymbol{C}: \left(\boldsymbol{2x^2+7x-3}\right)$ $\boldsymbol{D}: (\boldsymbol{-x^2-5x+6})$

31. $D + C$

32. $B - D$

33. $B \cdot C$

34. $A - C$

35. $D \cdot A$

36. $D + B$

37. List the criteria that make an equation <u>not</u> a polynomial in one variable. (3)

38. Is $3x^2 - 5x + 11$ a polynomial in one variable?
    a. If so, what kind of polymonial is it? Monomial Binomial Trinomial Polynomial

39. Is $5x^3 + x^{\frac{1}{2}}$ a polynomial in one variable?
    a. If so, what kind of polymonial is it? Monomial Binomial Trinomial Polynomial

Use the following equation to answer questions #40-41. $y = 2x^3 - x^2 - 13x - 6$

40. What is the degree of the equation and what does it tell you about the graph?

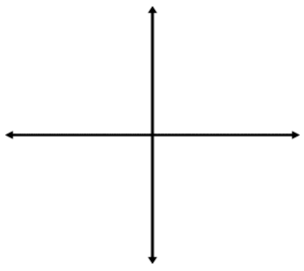
41. What two pieces of information do you need from the equation to determine the end behaviors of the graph?

Determine the End Behavior L: _____ R: _____

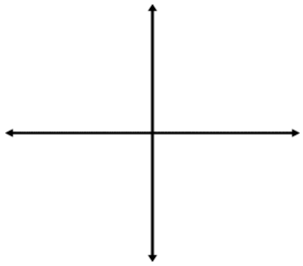
42. Determine the End Behavior of the following equations:

a. $y = -4x^7 + 8x^2 + 2x - 5$ L: _____ R: _____	b. $y = 5x^5 - 10x^3 + 1$ L: _____ R: _____
c. $y = 12x^4 + 3$ L: _____ R: _____	d. $y = -7x^{10} + 8x^6 - 14x^5 + 6x - 13$ L: _____ R: _____

43. Sketch a polynomial function with the following features:
    a. Degree 6
    b. 3 real roots
    c. Leading coefficient is positive
    d. Negative y-intercept



44. Sketch a polynomial function with the following features:
    a. Degree 3
    b. 3 real roots (including a double root)
    c. Leading coefficient is negative
    d. Negative y-intercept



For questions 45-48, divide.

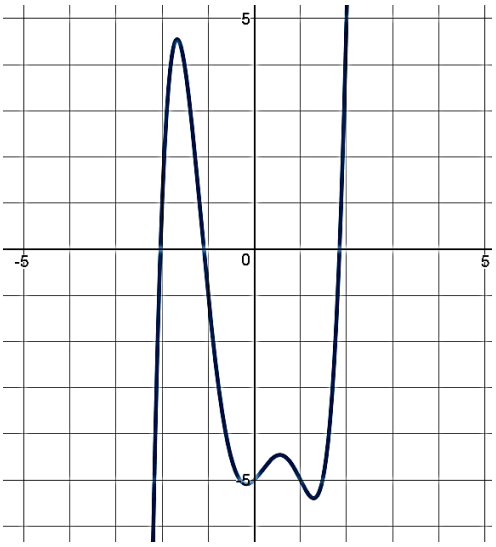
45. $\frac{8x^4-4x^2+x+4}{2x+1}$

46. $\frac{6x^3+5x^2+9}{2x+3}$

47. $(x^5 + 2x^4 + 33x^2 - 20)(x + 4)^{-1}$

48. $(6x^5 - 18x^2 - 120) \div (x - 2)$

49. Consider the polynomial function $P(x) = x^5 - 4x^3 + 2x^2 + 3x - 5$.



a. How many total roots does P have? Explain.

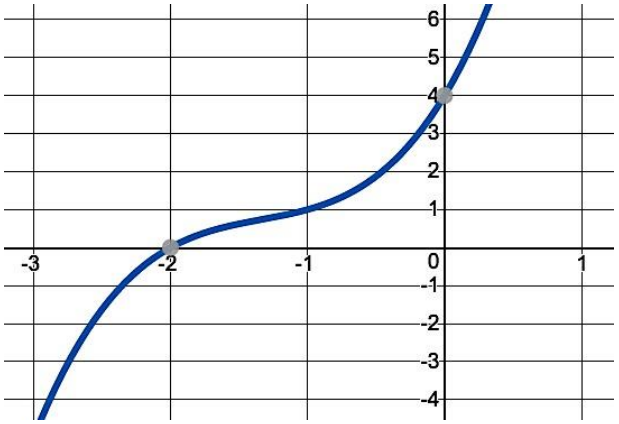
b. How many real roots does P have? Explain.

c. How many imaginary roots does P have? Explain.

d. State the following:

- Leading coefficient: positive or negative
- Number of Relative Maximum(s): _____
- Number of Relative Minimum(s): _____
- End Behavior: Left _____ Right _____

50. Consider the polynomial function $y = x^3 + 4x^2 + 6x + 4$.



a. Use the graph to find the real zero(es).

b. Use the remainder theorem to prove your answer(s) from part "a" are correct.

c. Use synthetic division and the quadratic formula to find the imaginary solutions.

51. Find all roots, both real and imaginary, of the polynomial. $y = x^4 - 2x^3 - 10x^2 + 86x - 195$

52. Find all roots, both real and imaginary, of the polynomial. $y = 3x^4 + 23x^3 + 8x^2 - 62x - 140$

53. You have a $9in \times 4in \times 11in$ box. You would like to increase each side of the box by the same amount so that the new volume is 12 less than 3 times the original.

    a. Write an equation to represent the situation.

    b. How many inches will you have to add to each side of the box?

    c. What are the new dimensions of the box?

54. You have a $19in \times 22in \times 26in$ box. You would like to decrease each side of the box by the same amount so that the new volume is 703 more than one-fourth the original.

    a. Write an equation to represent the situation.

    b. How many inches will you have to take from each side of the box?

    c. What are the new dimensions of the box?

For #'s 55-58, write a polynomial function in standard form that has the given zeroes.

55. $x = -4, x = 1, x = 5$ Assume "a"$= 1$

56. $x = 4i, x = 3, x = -3$ Assume "a"$= 1$

57. $x = 2i, x = 1$ passing through (2,6)

58. $x = 0,\ x = 2 + 2i$ passing through $(2, -32)$

For questions 59-66, perform the indicated operation and write an equivalent rational expression in reduced form (simplify). Don't forget to state restrictions.

59. $\frac{x}{x^2+9x+20} + \frac{-4}{x^2+7x+12}$

60. $\frac{2x+8}{3x^2+12x} + \frac{8}{3x}$

61. $\frac{5}{2x-3} - \frac{6x+21}{4x^2-9}$

62. $\frac{x^2}{x^2+2x-15} - \frac{30+x}{x^2+2x-15}$

63. $\frac{x-3}{x+4} - \frac{2x-34}{6x+24}$

64. $\frac{x^2-10x+24}{2x^2-5x-12} * \frac{x^2-81}{2x^2-12x}$

65. $\frac{x^2+7x}{3x} \div \frac{x^2-49}{3x-21}$

66. $\frac{\frac{x^2+2x-8}{x^2+4x+3}}{\frac{x-2}{3x+3}}$

For questions 67-70, solve. Check your solutions.

67. $\frac{x+2}{x-5} - \frac{45}{x^2-x-20} = \frac{5}{x+4}$

68. $\frac{x-4}{x-2} = \frac{x-2}{x+2} + \frac{1}{x-2}$

69. $\frac{2}{x+2}+\frac{x}{x-2}=\frac{x^2+4}{x^2-4}$

70. $\frac{9}{x-3}-\frac{x-4}{x-3}=\frac{1}{4}$

For questions 71-74, solve. Check your solutions.

71. $\sqrt{x-5}=\sqrt{2x-4}$

72. $5\sqrt[3]{x-1}-13=2$

73. $\sqrt{4x-5}+5=12$

74. $\sqrt{x-15}=3-\sqrt{x}$

75. Prove that $x = \sqrt{-3x + 4}$ has one solution.

Write the equation in exponential form.

76. $log_7 2401 = 4$

77. $log_2\left(\frac{1}{64}\right) = -6$

78. $log_{\frac{1}{3}}(\frac{1}{243}) = 5$

Write the equation in logarithmic form.

79. $5^4 = 625$

80. $3^{-3} = \frac{1}{27}$

81. $\left(\frac{1}{2}\right)^{-5} = 32$

Solve.

82. $81^{x-7} = 729$

83. $5^{x+8} = 11^x$

84. $log(8x + 19) - log3 = log(4x + 5)$

85. $lnx + ln(x-2) = ln48$

86. $3^x = 12$

87. $e^{2x} = 85$

88. $log_4(3x-5) = 3$

89. $log_2(x+15) + log_2 x = 4$

90. $\ln(7x) = 4$

91. $log_{14}625 = 4log_{14}x$

92. $log_2(12x) - log_2 3 = 7$

93. $\left(\frac{1}{5}\right)^{2x} = 125^{x-6}$

94. The Billy Idol fan club membership has been increasing at a rate of 83% annually since I helped start the club in 1995. At its origin, we had 9 members. How many members will we have in 2025?

95. Your grandfather gave you $25,000 for college. You decide to invest it for the next 5 years.
    a. What is the better choice?
- Fells Wargo, who compounds weekly, with an interest rate of 3%
- Dohn Jeere credit union, who compounds continuously, with an interest rate of 2.8%

    b. If you decided to choose Dohn Jeere, when would your money reach $30,000?

96. Your neighborhood is going downhill ever since your crazy, loud neighbors moved in back in 2013. At that point, your house was worth $124,500.
    a. If the value is decreasing at 2.4% each year, how much will it be worth when you try to sell it in 2017?

    b. If you decide to not to sell in 2017, when would you need to sell in order to keep the value of your house above $100,000?

97. You have $10,000 to invest at 1.5% interest. How much will you have after 20 years if you compound:

a. continuously?

b. monthly?

c. If you compound monthly, how long until you reach $20,000?

98. Find all roots, both real and imaginary, of the polynomial. $y = x^5 + x^4 - 7x^3 - 7x^2 - 144x - 144$

99. Find all roots, both real and imaginary, of the polynomial. $y = 4x^5 + 8x^4 - 21x^3 - 11x^2 + 20x - 84$